TO THE RIGHT WOR^pfull, the *Masters*, *Wardens*, and *Assistants*, of the *Trinitie*-House of DEPTFORD-STRAND.

WHen I first published this Work (*Right Wor:*) my chiefe ayme was for the benefit of Sea-men, many of whom are ignorant of the Latine-*tongue*, whereof this is but a *Translation*; the *Author* needeth not my *Commendations*, for his Workes doe *sufficiently testifie of him.* *And albeit since the first publishing hereof, the admirable Invention of the* Logarithmes *have been found out, by that never to be forgotten,* JOHN NEPER *Lord of* Merchistone, *vpon whose foundation Mr.* HENRY BRIGGES, *publique Professor of* Geometrie *in the* University *of* Oxford, *hath altered the same and made them more facile for all manner of* Arithmeticall *workes: Yet the ground of this* Trigonometrie *still*

remaineth, whose Rules are certaine and infallible as those of the Logarithmes; *whereof hereafter by Gods assistance I may write, if my many Imployments hinder not; In the meane time knowing that many Mariners have by their industrious labours reaped fruit by my former paines. I resolved, for their sakes to revive the same; and to Dedicate it to You who sit at the helme of their Government, to guide and direct them in their true Course; not doubting, but for the Workes sake, and in memory of* Mr. WALTER WHITING, *one of your Fraternity and my deare deceased Friend, You will accept of the weake labours of*

Yours, ever to be commanded:

R. H.

To the most Noble Prince, Lord FREDERICKE the Fourth, *Count Palatine* of *Rheine*, Chiefe *Sewer*, and Elector of the *Roman* Empire, Duke of *Bavier*, &c. his most gracious Lord.

MOst renowned Prince Elector, and Soveraigne Lord, If my whole Life had not been knowne to your most Noble Excellency, I might largely have excused my selfe: For that, I (being a Divine, *as one unmindfull of my Vocation) should not onely practice the* Mathematicks, *but also write publike Bookes of that kind. For I doubt not but many would maliciously calumniate these my Studies, but that they know your renowned Excellencie will bee ready to stand in my defence. And truely if I should bestow the time that I ought to spend in divine Meditations, in numbring of the* Starres; *I might bee worthily reprehended: But now, sithence I am conversant in these Studies, at such times when others are Idle; and to no other end but that I may readily and truly answer your Excellencie, who often questioneth mee of these matters: What is he that will not preferre my honest recreations before others sloath, or that can reprove your noble Excellencies forwardnesse in advancing profitable Sciences?* Abraham *the Patriarch is commended of* Iosephus, *because he gave light in the* Mathemati-

call Arts, *and trained up others in them. And in the Booke of* Daniel, *this is not the least, that he was instructed in all the wisedome of the* Chaldeans, *which chiefly consisted in learning Demonstrative. Neither are the workes of God set forth, lesse for the Divines, then for other sorts of men, that in beholding them, they may learne to admire the wisedome, feare the power, and magnifie the glory of God,*

And all these affections, doubtlesse, in a zealous man, are so much the more fervent, by how much the more he hath understanding of the works of God. Every foole in beholding the Sun, wonders at his brightnesse, the power of his heat, the swiftnesse of his motion, and the certainty of its course; but yet he knoweth not the forme and magnitude of the Sun, *and how long is his way that he daily maketh. If you shall say and demonstrate to him out of the rules of* Astronomy, *that the* Sun *is a round body 166 times greater then the* Globe of the Earth, *and that the* circuit of his daily motion *is more then* 4000000. German *miles he will leave wondring and stand amazed at so great secrets of* Nature, *crying out with* David; O Iehovah our God, how marvailous is thy Name in all the Earth! and what is man that thou which art the workman and maker of such things, shouldest bee mindfull of him? *Moreover, next to the secret operation of the spirit of God, it is to be deemed, that nothing doth make a man more meeke and gentle then the study of this* Heavenly Philosophy: *and how admirable and rare an ornament, O good God, is mildnesse in a* Divine? *and how much is it to be wished in this age that all* Divines *were* Mathematicians? *That is, men gentle and meeke. Howbeit, least any man mistaking me, should attribute too much to these Speculations, and in the meane time, neglect his duty; I must needs confesse that moderate and indifferent exercise in these Studies do hurt no man; so that their publike and conti-*

nually

nually reading doe not somewhat hinder them, who ought to preserve their whole strength both of body and mind, for the vndergoing of other labours : which when I had within this sixe Moneths, duly examined, I purposed with my selfe to write no more of this subiect, and I procured others of my ranck to giue ouer the same, For Lodovicus vives *saith truly,* the Wit not overmuch tyred is, more pregnant. *And Christ speaking grauely* : Let the dead bury their dead, but goe thou and preach the Kingdome of God. *This rule then let vs obserue, Yet because what I haue already written,* Most gracious Pr. Elector, *may not onely proue profitable to you, but also to many others : why should I suppresse the same, for thus much I presume I may say without boasting, that the* Doctrine of Triangles *was neuer yet of any man so plainely set forth, and the vse thereof in so many Arts, so familiarly explained : especially I am sure it will delight all those of ripe iudgment, when they shall see that by the Problems of the motion of the* Sun *and* Moone, *all the Heauenly motions (for the same reason is of the rest) may be found out without any helpe either of* Alphonsus *or the* Prutenick tables, *only by the* doctrine of Triangles, *and vulgar* Arithmaticke, *with the same ease, truth, and pleasure, as by tables far greater : wherefore I doubt not but your Excellency in time will take very much pleasure therein. For after your Highnesse hath learned throughly* Arithmetick, *not neglecting the grounds of* Geometry, *nothing can then hinder, why you may not attaine to this Science wherby your Highnes may gaine to your selfe for euer more then a Kingly name : For, by how much there are few Princes that vnderstand these things, by so much the more is their praise when they vnderstand them. And your Excellency knoweth that your worthy Vncle* Wil. Landgrave *of* Hessea, *my worthy Patron, though he excelled in other Arts, yet obtained he a more glorious name by the study of*

Astro-

Astronomy, *and it is well knowne that the memory of* Alphonsus *King of* Arragon *had long since been buried but that the* Tables of the Heauenly Motions *calculated by his care, and at his costs, are of necessary vse amongst the learned. Therefore let your Excellency account it a Kingly praise to imitate these thrice famous Kings and Princes; whereunto if I shall any way be able to giue assistance my care and industry (as your Highnes bounden seruant) shall neuer be wanting to accomplish your desires. For although, as I said before, I will not publikely treate of these Arts; yet if your Highnesse shall command me any more seruice in this kind, I will, and am bound to vndergoe the same for your Excellencies pleasure, sithence I (principally) and my whole family haue, after so many yeares seruice, and so many fauours receiued, beene highly rewarded For which fauours, because they are many more then my slender seruice can requite, I beseech God to vouchsafe the reward of them with the riches of his Grace, and evermore to blesse your Highnesse, together with your most Princely wife, and whole Progeny, with all Corporall and Spirituall Graces. Of which my desires and wishes, let this Dedication beare witnesse. Written in your highnesse Court* at Hagenbach *the* 12, *of September*, Anno Dom. 1599.

Your Excellencies most

humble deuoted Seruant:

B: PITISCVS.

THE FIRST BOOKE OF TRIGONOMETRIA:

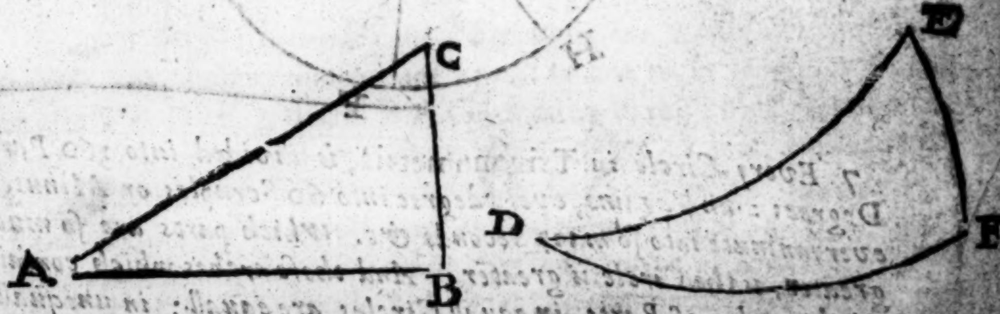
Written by BARTHOLMEW PITISCVS, of *Grunberge*; First written in Latine, and translated into English by *Ra: Handson*, Student in the Mathematickes.

Of the Nature and qualitie of Triangles.

I.

TRIGONOMETRIA, is the Doctrine of the measuring of Triangles:

2 A Triangle is a Figure comprehended of three sides, and three Angles, as are the figures A B C, and D E F.



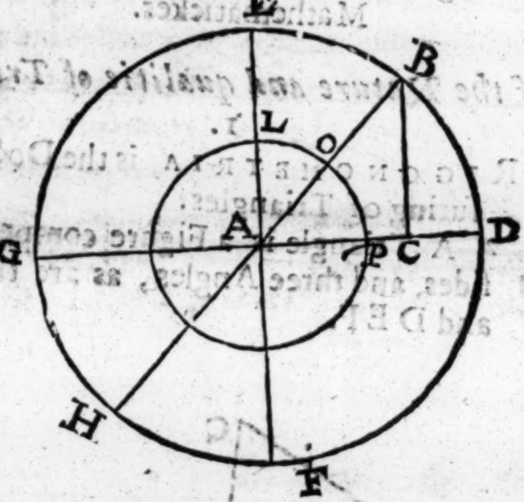
3 Every of the two sides are, the sides of the Angle, comprehended by them; the third is the Base. As the sides A B, and A C, are sides of the angle B A C. And B C, is the base of the said Angle.

Every

4 *Every side is said to subtend the Angle opposite unto it.* As the side A B, subtendeth the angle A C B : The side A C, subtendeth the angle A B C, and the side B C, subtendeth the angle B A C.

5 *The greater sides subtend the greater Angles; and therefore the lesser sides the lesser angles, and equall sides equall angles.* The truth of the Theorem is manifest of it selfe; yet it is demonstrated in the 18 and 19 *Probl.* of the first booke of *Euclide*, and in the 42 and 43 *Prob.* of the 3. booke of *Regiomontanus* : It is also plainly confirmed by the second Axiome of the 3. and the third of the 4 booke following.

6 *The measure of an Angle, is the arch of a Circle, described from the point of the Angle, and intercepted betweene the two sides (of that Angle) increased.* As in the triangle A B C, the measure of the angle B A C, is the arch O P, or B D.



7 *Every Circle in* Trigonometria, *is divided into 360 Parts, or Degrees : and againe, every degree into 60 Scruples or Minutes, and every minute into so many Seconds, &c. Which parts are so much the greater, as the Circle is greater; And those arches which containe the same number of Parts, in equall Circles, are equall; in unequall Circles, they are said to be like Arches* : As the arches B D, and G H, are equall. But the arches B D and O P, are like arches : For example; As B D is 40 parts in the great Circle E B D; so is O P 40 parts in the lesse circle L O P, &c.

8 Then

8 *Then a Quadrant of the said Circle, is the Arch of 90 parts.*

9 *The Complement of an arch, lesse then a Quadrant, is, so much as that arch wanteth of 90 parts.* As the Complement of the arch B D 40 parts, is the arch B E 50 parts. And in like manner.

10 *The excesse of an Arch greater then a Quadrant, is so much as the said arch is more then 90 parts.* As the excesse of the arch G E B 140 parts, is the arch E B 50 parts, more then a Quadrant.

11 *A Semi-circle, is an Arch of 180 parts.*

12 *The complement to a Semicircle of an arch, lesse then a Semicircle, is so much as that arch wanteth of 180 Parts.* As the complement of the arch G E B 140 parts, is the arch B D 40 parts.

13 *The opposite angles made by crossing of two Lines are equall.* As the angles B A D and G A H, are equall; and likewise the angles G A B and H A D are equall: So also is it in Sphearicall angles. The truth of the *Theorem* appeareth of it selfe; Yet it is demonstrated in the 15 Pro. of the first booke of *Euclide*, speaking of right Lines mutually cutting one another.

14 *An Angle, is either right or oblique.*

15 *A right angle, is that whose measure is a Quadrant.*

16 *An oblique angle, is either obtuse or acute.*

17 *An obtuse angle, is that whose measure is more then a Quadrant*, as B A G.

18 *An acute angle, is that whose measure is lesse then a Quadrant*, as B A D.

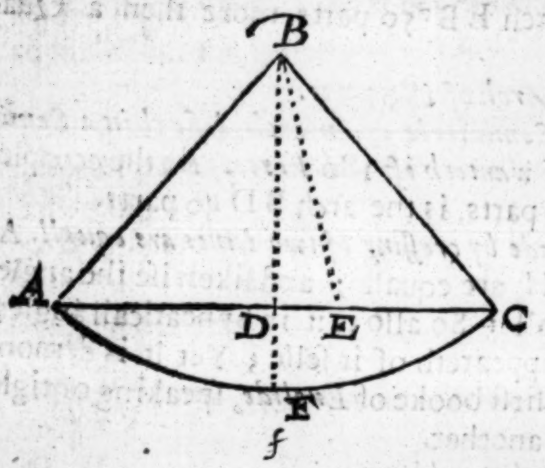
19 *The Complements of angles, are said to bee as the complements of Arches.*

20 *All Angles comming together upon one Line (drawne out at length) being taken together, are equall to two right Angles.* As the angles B A D, E A B and E A G meeting in the point A, upon the line G D; are equall to the two right angles G A E, and E A D, by the opperation.

21 *Therefore if two Oblique angles meet upon the said right Line, drawne out at length; the one is the Complement of the other, to two right angles.* As the angle B A D, is the complement of the angle G A B, to two right angles.

22 *A Triangle first of all, hath some of the sides equall, or else all the sides unequall.*

23 *If a Triangle have some of the sides equall, a perpendiculer let fall from the meeting of the equall sides, cutteth the base and the angle, opposite to the base, into two equall parts, and contrarily.* As in the Triangle of two equall sides A B and B C; the perpendiculer B D, cutteth the base A C, and the angle A B C, opposite to the Base, into two equall parts. It cutteth the base A C, into two equall parts; because if it should not cut it into two equall parts, but should fall without the middle point D, that is in E; it should not bee perpendiculer, and so not the shortest Line, betwixt the point B, and the right line A C. Also it cutteth the angle A B C, opposite to the base, and his measure A F C, into two equall parts; because the angles are as the sides, by the Fifth hereof.



24 *A Triangle of some equall sides, is either equicrurall or equilaterall.*

25 *An equicrurall Triangle, is that which hath only 2 equall sides.*

26 *An equicrurall Triangle, is equiangled at the base and contrary,* by the fifth hereof.

27 *An equilaterall Triangle* (so called through the excellency thereof) *is that, that hath all three sides, equall one to another.*

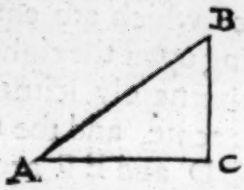
28 *An equilaterall Triangle, is equiangled, and contra:* by the fifth hereof.

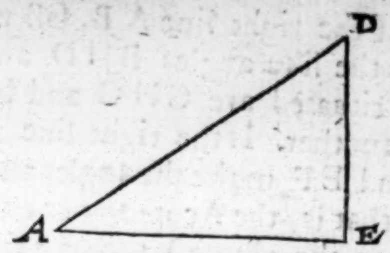
29 *Moreover, a Triangle is Right angled, or Oblique angled.*

30 *A right angled triangle, is that, that hath one Right angle.*

31 *In right angled Triangles, having only one right angle, the subtendent to the right angle, is commonly called the* Hipothenusa: *but the sides including the Right angle, are called the Perpendicular, and the Base (at pleasure.)* As in the Triangles A B C and A D E, the sides A B and A D, are the *Hipothenusas*; B C and D E, the *Perpendiculers*; A C and A E, the *Bases*; or contrarily, A C and A E, are the perpendiculers, and B C and D E, are the bases.

32 *An*





32 *An Oblique angled triangle, is that which hath all the angles oblique.*

33 *An oblique angled Triangle, is either obtuse angled, or acute angled.*

34 *An Obtuse angled triangle, is that, that hath but one obtuse angle.*

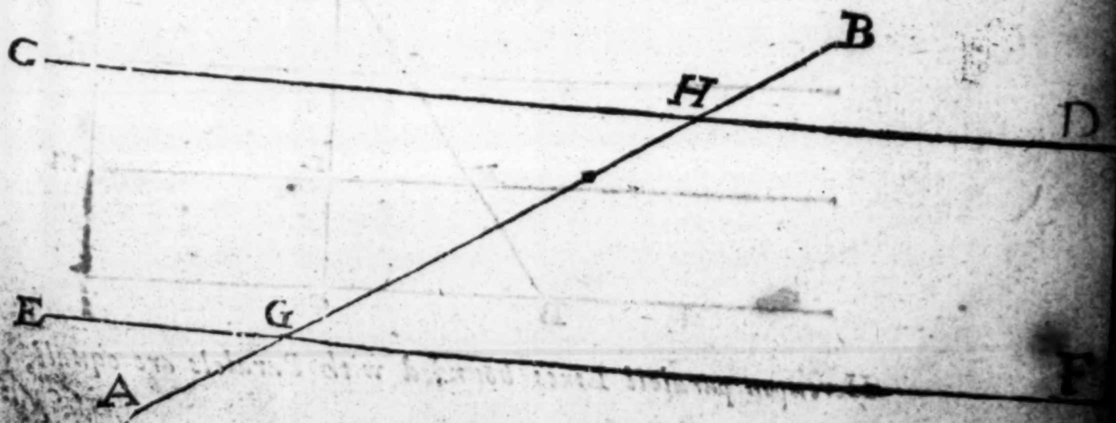
35 *An Acute angled triangle, is that, that hath all the angles acute.*

Lastly, *a Triangle is either Plaine or Sphearicall; plaine in a Plaine, and Sphearicall upon the Globe.*

37 *The sides of a Plaine triangle in* Trigonometria, *are right Lines onely.*

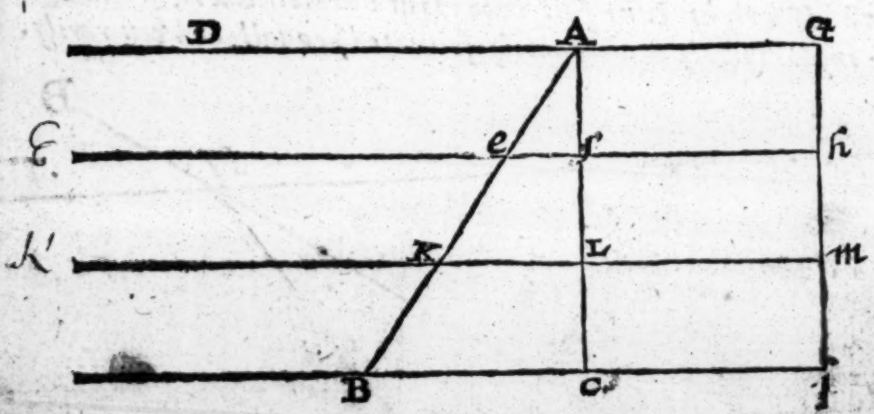
Touching right Lines, for the better understanding of *Trigonometria*: It is necessary to know these Theorems following.

38 *If a right Line fall upon right Paralell lines; It maketh the like angles (likely or alternately scituated) equall, and contrarily.*



As if the right line A B, fall upon the paralels C D and E F, it maketh the like angles B H D and B G F. Also the angles alternately scituated, are C H G and H G F, &c. which are equall all one to another. If the right line A B, falling upon the right Lines C D and E F, make the angles alike, and alternately scituated equall (that is, the Acute angles equall to the acute, and the Obtuse angles to the obtuse) then the right lines C D and E F are Paralels. It is the 29 of the first of *Euclide*. The naturall reason; For if A B be a right line, the right lines C D and E F, cannot be equally distant one from another; unlesse they incline to the right line A B with equall angles. *From hence may be gathered, If many right Lines bee perpendiculer to one right line, they are Paralell one to another.* As the right lines C D and E F, are paralell one to another; because they are Perpendiculer to the one and the same line D F.

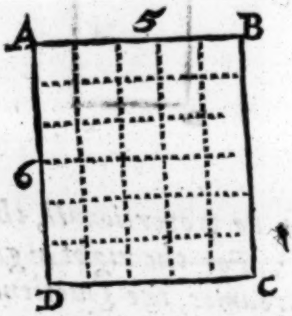
39 *If many right Lines are cut by divers other right lines, paralell one to another; the Intersegments are proportionall.* As if the two right lines A B and A C, are cut by the paralels E H, K M, and B I. I say the intersegments A E and A F, and likewise E B and F C, are proportionall one to another: That is to say; If A E, bee the $\frac{1}{3}$. part of the right line A B? A F also shall be $\frac{1}{3}$. part of the right line A C, &c. The reason is, because the right line E H, cutteth off $\frac{1}{3}$. part from the whole space D G I B; and therefore from all the Lines drawne through that space.

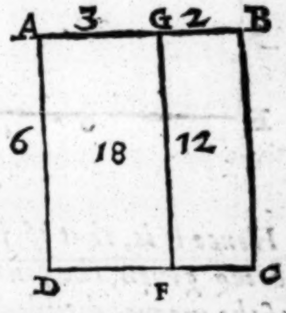


Hereupon paralell Lines bounded with Paralels are equall, and contra-

contrarily: As the paralels A F and G H, bounded with the paralels A G and F H, are equall :. For sithence the whole lines A C and G I, are equall; also of necessity, A F and G H being $\frac{1}{3}$ part thereof, are also equall.

40 *If two right Lines be multiplied in one another; there is made thereof a right angled Quadrangle.* As if the two right lines A B and A D, be multiplied one in another; thereof is made the Quadrangle A B C D. If then A B be 5 foot, and A D 6; the whole quadrangle A B C D, shall be 30 square feet, as appeareth by the pricked lines in the *Diagram*.

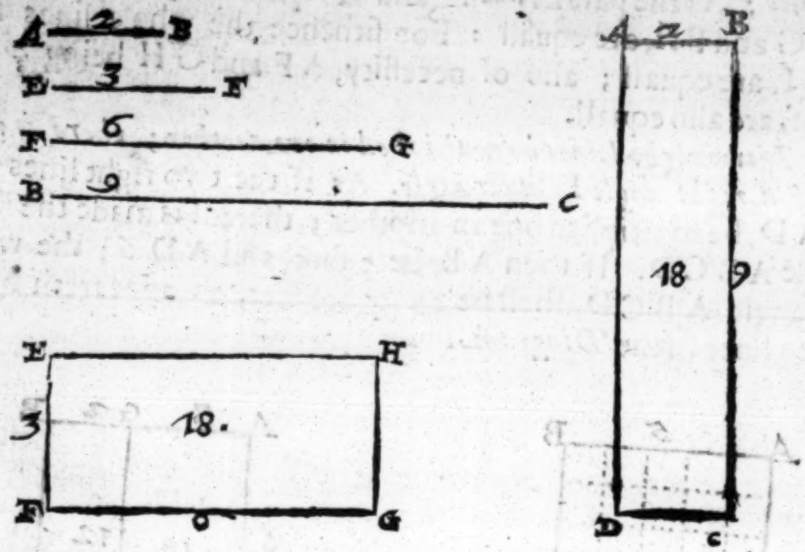




41 *The right angled Figures made of one of the whole lines of one side of the figure, and the Segments of the other side thereof added together, are equall to the right angled Figure made of both the whole Lines.* As the right angled figures made of the whole line A D 6, and the Segments A G 3, and G B 2; that is, the right angled figures A G F D 18, and F G B C 12, added together, are equall to the right angled figure A B C D 30, made of both the whole lines A D 6, and A B 5.

42 *If foure right Lines are proportionall* (that is, as the first to the second, so is the third to the fourth) *the right angled Figure made of the two meanes, shall be equall to the right angled Figure made of the two extreames.* As if there be foure Proportionals, A B 2, E F 3, F G 6, B C 9 feet, the right angled Figure made of the two meanes E F, and F G; that is, the right angled figure E F G H, is equall to the right angled figure made of the extreames A B, and B C; that is, to the right angled figure A B C D. For as twice 9, is 18; so is three times 6, 18.

1 Hence



1 *Hence it is, that if foure right Lines be proportionall, three of them being given, the fourth also is given ; For the right angled figure of the meanes divided by one of the extreames, the Quotient is the other extreame.* As if it were said :

As 2, to 3. So is 6, to 9.

The right angled figure made of 3 and 6, that is 18, divided by the first extreame 2, the Quotient is the last extreame 9, &c.

And this is the reason, why in the *Rule of Proportion*, commonly called the *Rule of* 3 ; the two latter tearmes are multiplied together, and that Product divided by the first, *viz.* Because the product of the Multiplication of the second and third tearmes, which divided by the first, shewes the fourth ; For *Division* and *Multiplication* produce one another mutually : and it is nothing materiall in the worke, whether of the two meanes you put in the second or third place. For either, you may say ;

As 2, to 3. So 6, to — &c. *Or*,
As 2, to 6. So 3, to — &c.

Although the proportion of the first, to the second, and of the third, to the fourth, bee one in the former, and another in the latter placing of the tearmes, yet you shall find the

same

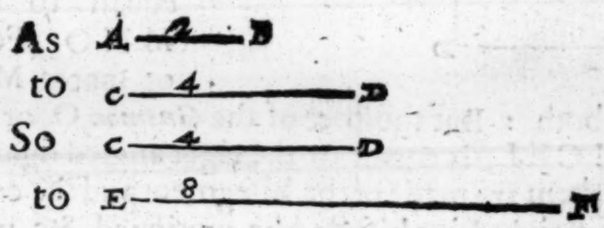
ſame anſwer in both; becauſe it is all one whether you multiply 3 by 6, or 6 by 3, &c.

2 *Hence alſo it is, that equall right angled Figures have their ſides reciprocally proportionall.* That is in equall right angled figures, as the leſſer ſide of the firſt right angled figure, to the leſſer ſide of the ſecond right angled figure: So is the greater ſide of the ſecond to the greater ſide of the firſt right angled figure. And *Contra*: As in the equi-rectangled figures ABCD, and EFGH, appeareth.

As $\underset{2}{\underline{AB}}$, to $\underset{3}{\underline{EF}}$. So is $\underset{6}{\underline{FG}}$, to $\underset{9}{\underline{BC}}$, &c.

The cauſe is manifeſt by the laſt *Diagram* afore-going.

43 *If three right Lines bee proportionall*, (that is, if as the firſt to the ſecond, ſo is that ſecond to the third:) *the Square made of the meane is equall to the Oblong made of the extreames.* For that the Meane is twice put, after this manner.

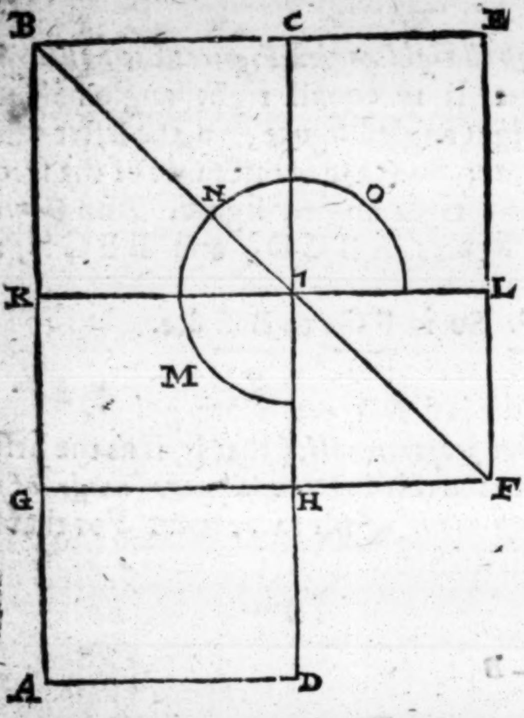


It is all one as if they were 4 Proportionals: Therefore whatſoever hath beene ſaid of foure proportionals, wee are alſo to underſtand of three proportionals.

44 *If a right Line being cut into two equall parts, be continued out at length; An oblong made of the line continued, and the line of Continuation; is equall to a Square made of a right line (of one of the Biſegments, and the line of continuation added together) leſſe by the ſquare of the ſaid biſegment by the 6 Prob.* 2 Euclide.

Let AK be a right line cut into two equall parts, in the point G, and continued to the point B: And let BC be equall to the continuation KB; and thereof let bee made the oblong ABCD.

Moreover, let the ſquare GBEF, be made of the right Line GB,



GB, which is equall to one of the Bisegments GK, and the Line of continuation KB, added together: From which square (by the right lines KL and CH,) let the square of the bisegment ILFH, bee cut off, that the *Gnomon* MNO, may remaine.

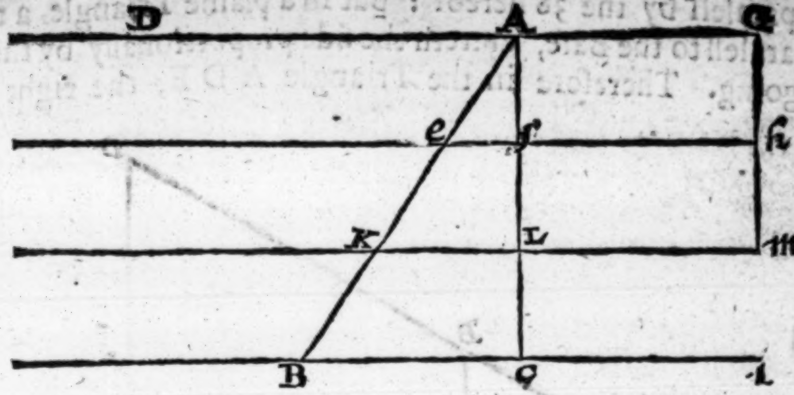
I say, that the Oblong ABCD, is equall to the square GBEF, lesse by the square ILFH, or which is all one: I say the oblong ABCD, is equall to the *Gnomon* MNO. For the figures or spaces M and N, are common to both: But the space of the *Gnomon* O, or the right angled figure ICEL, is equall to the right angled figure GHDA For both of them are made of the Bisegment and the continuation Therefore if a right Line bisected be continued, &c. which was to be demonstrated.

And thus much of right Lines; as of the sides of plaine Triangles, I have thought good to set downe. Now I will returne to plaine Triangles themselves.

45 *In a plaine Triangle, a line drawne Paralell to the Base, cutteth the sides thereof proportionally.*

As in the plaine Triangle ABC. If KL be paralell to the base BC, it cutteth off from the side AC $\frac{1}{3}$. part; and also it cutteth from the side AB $\frac{1}{3}$. part, by the 39 hereof. And so they shall bee proportionall.

As AB, to AC. So is AK, to AL. Also, —
As AK, to BK. So is AL, to CL. Also, —
As AK, to AL. So is KB, to LC.



46 *If divers plaine Triangles are compared together ? Equiangled Triangles, have their sides about the equall Angles proportionall, and Contra:* by the 4 *Pro. 6 Euclide.*

☞ This *Theorem* is the chiefe ground of *Trigonometria:* Therefore above others, it is to be diligently explained and Noted.

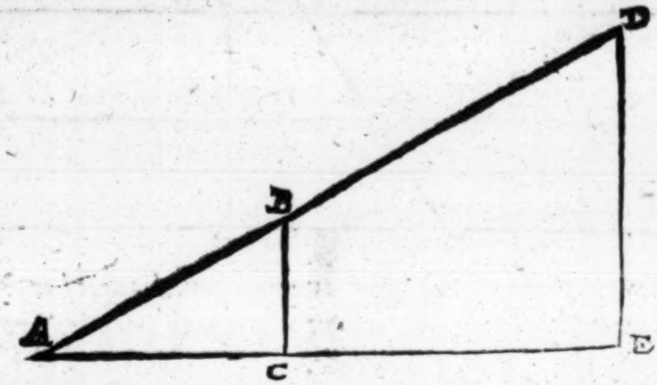
The Declaration. Let A B C and A D E, be two plaine equiangled Triangles, so as the angles at B and D, at A and A ; and also at C and E, bee equall one to another: I say, their sides about the equall angles are proportionall ; that is,

1 As A B, to B C. So is A D, to D E:
2 As A B, to A C. So is A D, to A E.
3 As A C, to C B. So is A E, to E D:

The Demonstration. For, because the angles B A C and D A E, are equall, by the *Pro:* Therefore if A B bee applyed to A D. A C shall of necessity fall in A E, and by such applications, such a figure shall be made.

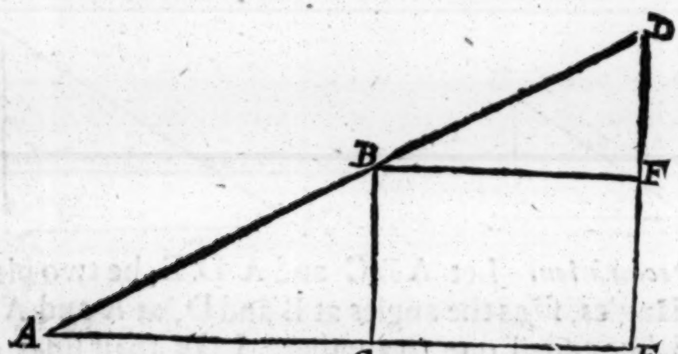
In which figure, because that A B and A C, doe meet together, and also the angles at B and D, and at C and E, are equall ; by the *Pro:* Therefore the other sides B C and D E, shall be of ne-

cessity paralell by the 38 hereof: but in a plaine Triangle, a right Line paralell to the Base, cutteth the side proportionally by the last afore-going. Therefore in the Triangle A D E, the right line



B C, being Paralell to the base D E, cutteth the sides A D and A E proportionally; and so

As A B, to A D. So is A C, to A E.



Moreover, by the point B, let the right Line B F, be drawne paralell to the base A E, and it shall cut the other two sides D E and D A, proportionally in the points B and F; by the same last afore-going. And then the proportion shall be,

As A B, to A D. So is F E, to D E. *Or, which is all one,*

As A B, to A D. So is B C, to D E.

For F E and B C, are equall by the 39 hereof. Besides, sithence they are;

As A B, to A D. So A C, to A E.

And so B C, *to* D E, *they shall be also*;

As A C, to A E. So B C, to D E.

For

For what things are agreeable to one third, are agreeable also to one another; Therefore generally,

1 As A B, to A D. So is B C, to D E.
2 As A B, to A D. So is A C, to A E:
3 As A C, to A E. So is B C, to D E.

Lastly, because it is not materiall to the worke, whether of the meane proportionall tearmes you place in the second or third place; by changing of these places, they shall be,

1 As A B, to B C. So is A D, to D E:
2 As A B, to A C. So is A D, to A E.
3 As A C, to B C. So is A E, to D E.

And so plaine equiangled Triangles (as these here A B C, and A D E, are) have their sides, comprehending the equall angles proportionall, which was to be demonstrated.

The illustration by Numbers. Let A B be 5 feet: A D 10, D E 06, and it is demanded how many feet is B C? Answer 3. For,

As { A D 10 to D E 06. So is A B. 05.

$\frac{30}{10}$ (to 3: B C.

Let *A C*, be 4 feet; *B C*, 3. *D E*, 6. and it is demanded, how many feet is *A E*? Answer 8. For,

As { B C 3 to A C 4. So is D E. 6.

$\frac{24}{3}$ (to 8. *A E*:

47 *If divers plaine Triangles be compounded, and be cut with right lines Paralels, the intersegments are proportionall*: As for Example. If the two Triangles *E A F* and *F A G*, bee compounded

and be cut with the right paralell lines B C D and E F G, their intersegments are proportionall.

As B C, to E F. So is C D, to F G. Or,

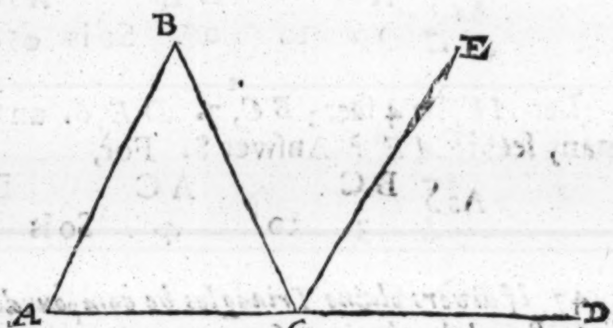
As B C, to C D. So is E F, to F G, &c. by the 39 hereof, or by the last precedent: For the triangles A B C and A E F, are equiangled by the 38 hereof; because B C and E F, are paralels: Therefore,

As A C to A F. So is B C to E F, by the last afore-going; but by the same.

As A C, to A F. So is C D to F G; and those that are agreeable to a third, are also agreeable to one another; therefore they are also

As B C, to E F. So is C D, to F G, &c.

48 *If any side whatsoever of a plaine Triangle be continued, the outward angle made by that continuation, is equall to the two inward opposite angles.* As if in the plaine triangle A B C, the side A C be continued to D; the outward angle B C D, shall be equall to the two inward opposite angles B A C and A B C. For if from the point C, were drawne the right line C E, paralell to the right line *A B*; the outward angle *B C D*, shall be compounded of the angles E C D and ECB. But the angles ECD and ECB, are equall to the two inward opposite angles *B A C* and *ABC* (that is to say, the angle *E C D*, to the angle *B A C*, and the angle *B C E* to the angle *A B C*) by the 38 hereof; because of the paralels *A B* and *C E*. And therefore the angle *B C D*, is equall to the two inward opposite angles *B A C* and *A B C*, which was to be demonstrated.



49 *The three angles in a plaine Triangle, are equall to two right angles.* As in the plaine triangle A B C, of the former figure; I say, the 3 angles A B C, BAC, and A C B, are equall to 2 right angles. For the angles meeting in one point, in one and the same Line, are equall to two right angles, by the 20 hereof. But the three angles

A B C,

A B C, *B C A*, and *B A C*, are equall to the three angles meeting in the point C, vpon the same line *A D*. For the angle, B C A, is common to both, and the angles E C D, and E C B, are equall to the angles *B A C* and *A B C*, by the last afore-going. Therefore the 3. angles, *A B C*, *B C A*, and B A C, are equall to two right angles, which was to bee demonstrated. Hence is it, that

1 *In a plaine Triangle, there can bee but one right, or one obtuse angle.*

2 *And one angle being right or obtuse, the other two are necessarily acute angles.*

3 *And the third angle, is the complement of any of the other two, to two right angles. Hence*

4 *Lastly, if two Triangles are equiangled, in two of their angles, they are wholly equiangled.*

50. *In a right angled plaine Triangle, the sides including the right angle, are equall in power, to the Hypothenusa.* By the last but one *Pro.* 1. *Euclide.*

The declaration: In the right angled plaine Triangle *A B C*, right angled at *B*. I say the sides A B and A C, including the right angle A B C, are equall in power to the hypothenusa A C; that is, the squares of the sides A B and B C, to wit, the squares A L M B and B E D C, added together, are equall to the square of the hypothenusa A C; to wit, the square A C K I.

The demonstration: For if from the right angle B, bee let fall the perdendiculer B F G, then out of the square A C K I, is made the two oblongs A F G I and F C K G, which are equall, to the square B E D C, and that other to the square A L M B. And therefore the square A C K I, compounded of those two oblongs, is equall to the two squares, A L M B and B E C D.

But that the two oblongs, A F G I and F C K G, are equall to the two squares, *A L M B* and *B E D C*, is to be prooued every one in particuler. And first of the oblong *A F G I*, it is thus prooved.

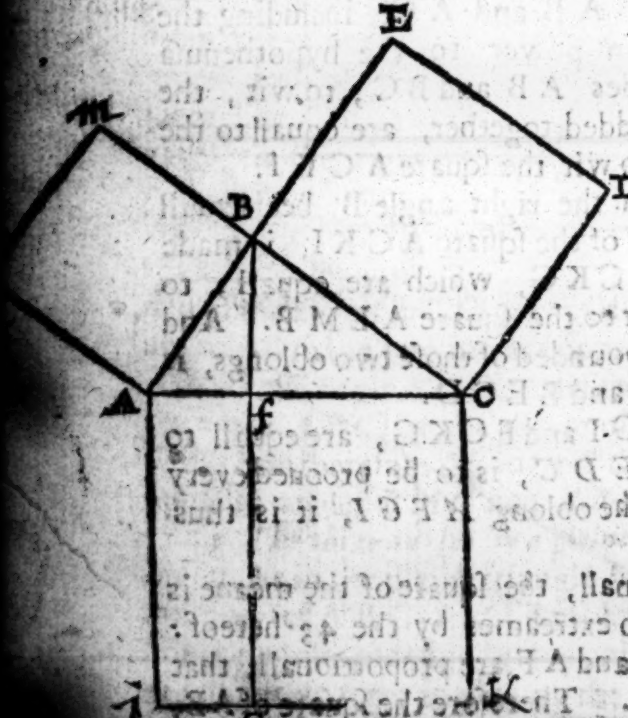
If three right Lines be proportionall, the square of the meane is equall to the oblong made of the two extreames by the 43 hereof: But the three right lines AI, A B, and A F, are proportionall; that is, as A I to A B. So is A B to A F. Therefore the square of A B, is equall to the oblong, made of A I, and A F,

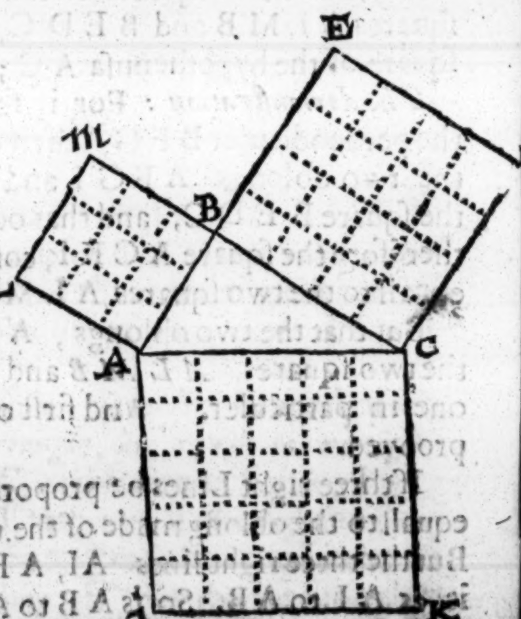
The *Minor* is proved; for the Triangles A B C and B A F, are equiangled, because of the common angle at A, and the two right angles B and F, by the fourth Consectarie of the 43 hereof: Therefore by the 46 hereof, as A C (equall to A I) to A B. So is A B, to A F.

In like manner it is altogether proved, that the Oblong F C K G, is equall to the square B E D C. For the Triangle A B C, and B C F, are equiangled; because of their common angle at C, and the two right angles at B and F, by the fourth Consectarie of the 49 hereof.

Therefore by the 46 hereof; as A C to B C, so is B C to F C.

And so by the 43 hereof; the square of B C, is equall to the oblong made of the lines A C, to K C, and F C. Therefore in a right angled plaine Triangle, the sides including the right angle, are equall in power to the Hypothenusa, which was to bee Demonstrated.





Commentary.

By a more mechanicall way, this *Pro:* may bee demonſtrated, *viz.* Let A B C be a Triangle, right angled at B, and let A B, be 3. B C, 4. and A C, 5 feet; let every ſide be ſquared, and let every Square bee diſtinguiſhed into ſquare feet, by the pricked Lines: and you ſhall ſee the ſquare of the Hipothenuſa A C, to have in it ſo many ſquare feet, as the ſquares of A B and B C, taken together.

Conſectarie.

Therefore in a right angled plaine Triangle, any of the two ſides being given, the third may be ſaid to be given. As if the two ſides including the right angle A B, and B C be given: *viz.* 3. and 4. their Squares 9, and 16. being added together, is 25; the ſquare Root thereof being extracted, the Hypothenuſa A C, ſhall be found 5 parts.

Contrarily, if the Hypothenuſa 5, and one of the ſides, including the right Angle 3, bee given; ſubtract the Square of 3, from the ſquare of 5; That is, the ſquare 9, being ſubtracted from the ſquare 25; and out of the Remainder being 16, the ſquare Root being Extracted; the other ſide including the right Angle, ſhall bo found 4 parts.

Commentaries, about the Extraction of the Square Root.

1 If after the Extraction of the ſquare Root of any number, any Fractions ſhall remaine, put for Denominator under thoſe Fractions, the Root doubled with 1, added thereunto; after this manner, —

3
12 ($3\frac{3}{7}$.

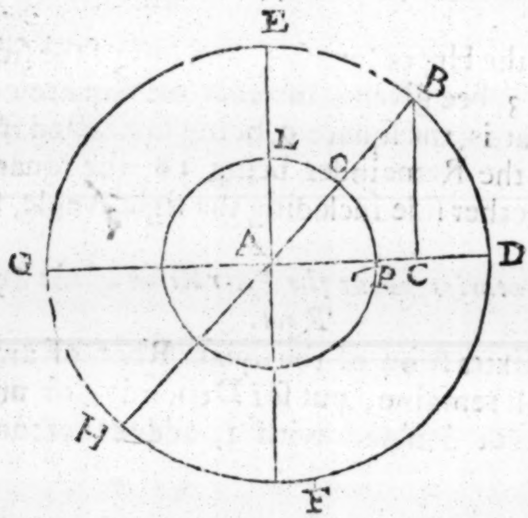
2 The Root which hath theſe Fractions adjoyned, is never exactly true. For the true Root multiplied in it ſelfe, ought to produce the Number where-out it was Extracted, without any difference. But if you multiply the Root, $3\frac{3}{7}$. in it ſelfe; that is, if you multiply $3\frac{3}{7}$. by $3\frac{3}{7}$. you ſhall not produce the true Number 12, out of which $3\frac{3}{7}$. was extracted; but onely $11,\frac{37}{49}$. Concerning which, ſee *Ramus* in his Elements of *Geometry*, *Elem.* 8. *Lib.* 12. And *Lazarus Schoner*, in his Comment upon *Ramus* Arithmetick.

11 *In a plaine right angled Triangle, the ſides including the right angle,*

angle, are for the most part irrationall to the Hipothenusa: that is, inexplicable in an exact number, of what quantity soever. The cause appeareth by the second commentarie next before going.

52 *In a plaine right angled triangle, the one of the acute angles, is the complement of the other*, by the 49. hereof. It is very easily prooved in this manner.

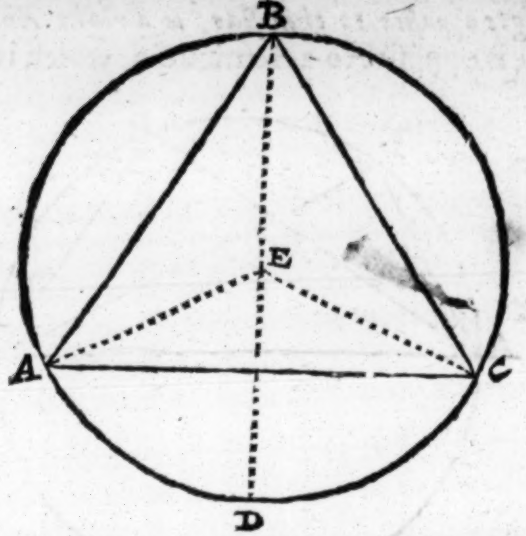
In the plaine Triangle A B C, right angled at C, the one of the acute angles A B C, is equall to the angle B A E, by the 8. hereof; because of the paralels, E A and B C. But the angle E A B, is the complement of the angle B A C, by the worke; Therefore is the angle A B C, the complement of the angle B A C.



53 *If a plaine Triangle be inscribed in a Circle, the angles opposite to the circumference, are $\frac{1}{2}$. as much as that part of the circumference opposite to the Angles*, As if in the Circle A B C, the circumference BC, be 120. deg. then the angle B A C, opposite to the circumference A B, shall be 60. deg. The reason is.

Because the whole circumference A B C, is 360. deg. by the 7. hereof. But the three angles of the Triangle A B C, inscribed in the Circle, are 180 degr. by the 49. hereof: Therefore as every arch is the $\frac{1}{3}$. part of 360 deg. so every angle opposite to that arch, is the $\frac{1}{3}$. part of 180. degrees.

It



It is more plainely thus demonstrated: *As for Example*, of the angle A B C. From the said angle A B C, let the Diameter B E D; be drawne through the whole plaine of the circle.

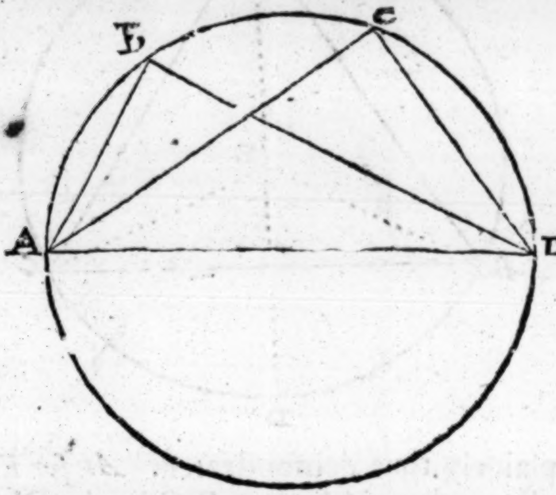
And from the center E, to the circumference A B C D, let the two *Radii* E A, and E C be drawne: I say, the divided angles A B D and D B C, are the ½. of the angles divided A E D and B E C. For the angles A B E and B A E, are equall by the 5 thereof; But the angle A E D, is equall to the angles A B E and B A E, added together, by the 48 hereof. Therefore the angle A E D, is double to the angle A B D.

In like manner; the angles E B C and E C B, are equall by the 5. th hereof; and to both these together, is the angle D E C equall, by the 48 hereof. Therefore the angle D E C is double to the angle D B C.

Then because the parts of the angle A E C, are double to the parts of the angle A B C. Therefore also the whole angle A E C, is double to the whole angle A B C. And thereupon the angle A B C, is ½. of the angle A E C, and consequently ½. of the arch A D C, which is the measure of the angle A E C. The same proofe is of the rest. If therefore a plaine Triangle be inscribed in a circle, the angles opposite to the Circumference are ½. of that part of the circumference opposite to the angles, which was to bee Demonstrated. *Hence it is, that*

1. *If*

1 *If the side of a plaine Triangle, inscribed in a Circle, be the Diameter; the angle opposite to that side, is a right Angle:* That is, 90 deg. for that it is opposite to a Semicircle, which is 180 deg.

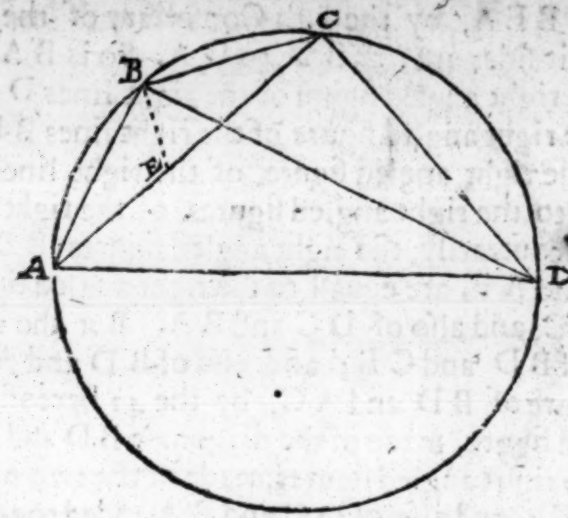


2 *If divers Triangles right lined, bee inscribed in the same Segment of a Circle, upon one base; the angles in the Circumference are equall.* As the two Triangles A B D and A C D, being inscribed in the same segment of the Circle A B C D, upon the same Base A D, are equiangled in the points B and C, falling in the circumference; For the same arch A D, is opposite to both those Angles; that is, to the angle A C D, and also to the angle A B D.

54 *If two plaine Triangles, inscribed in the same segment of a Circle, upon the same Base, bee so joyned together in the top, (or in the angles, falling in the Circumference) that thereof is made a Quadrilaterall figure, intersected with Diagonals; The right angled figure made of the Diagonals, is equall to the right angled Figures (added together) made of the opposite sides.* Ptolemie and Copernicus.

The Declaration. Let A B D and A C D, be two Triangles, inscribed in the same Segment of the circle A B C D, upon the same base A D, so joyned in the top by the right line B C, that thereupon is made the foure-sided figure A B C D. I say, that the right-angled figure made of the two Diagonals A C and B D, is equall to the right-angled figures together, made of the opposite sides

AB



AB and DC, and also of the sides BC and AD.

The Demonstration.

For if at the point B, you make the angle ABE, equall to the angle DBC, and so you cut the Diagonall AC, into two parts by the right Line BE, at the point E. It is manifest, that the right angled figures of BD and EC; and also of BD and EA, are equall to the right angled figures, made of BC and DA; and also of CD and AB. For if foure right Lines be proportionall, the right angled figure made of the meanes, is equall to the right angled figure made of the extreames by the 42 hereof. But the foure right Lines BD, DA, BC, and CE, are proportionall. For because the Triangles ABD and BCE, are equiangled, because of the equall angles BCA and BDA, by the 2d. *Consect.* afore-going; also because of the equall angles ABD and EBC (which are equall) for that the same EBD, is added to the equall angles ABE and DBC; and lastly, because of the equall angles BEC and BAD, by the 4 *Consect.* of the 49 hereof. Therefore their sides are; As BD, to DA. So is BC, to CE. In like manner, the foure right lines BD, DC, BA, and AE, are proportionall.

For because the Triangles BDC and BAE, are equiangled, because of their equall angles BDC and BAE, by the second *Consectary* afore-going. Also because of their equall angles DBC and ABE, by the Proposition; and lastly, because of the equall angles

gles B C D and B E A, by the 4.th Consectary of the 49 hereof.

Therefore their sides are; as B D, to D A. So is B A, to A E.

Therefore the right angled figure of the right lines D A and B C, are equall to the right angled figure, of the right lines B D and C E. And likewise the right angled figure, of the right lines D C and B A, are equall to the right angled figures, of the right lines B D and A E. And contrarily, the right angled figures B D and C E; and also B D and A E, are equall to the right angled figures, made of D A and B C, and also of D C and B A. But the right angled figures, made of B D and C E; and also of B D and A E, are the right angled figure of B D and A C, by the 41 hereof. Therefore the right angled figure, made of the diagonals B D and A C, are equall to the two right angled figures, made of the two opposite sides of D A and B C; and also of D C and B A added together, which was to be demonstrated.

Consectary.

Therefore in a Quadrilaterall *figure inscribed in a Circle, and intersected with Diagonals, and so consisting of 6 right Lines: Any 5. of them being given, the 6. is also given.* You have most excellent Examples hereof in the second Booke. *Pro.* 32, 33, 35, 36, 37, 38.

Part 2d.

And thus much of plaine Triangles.

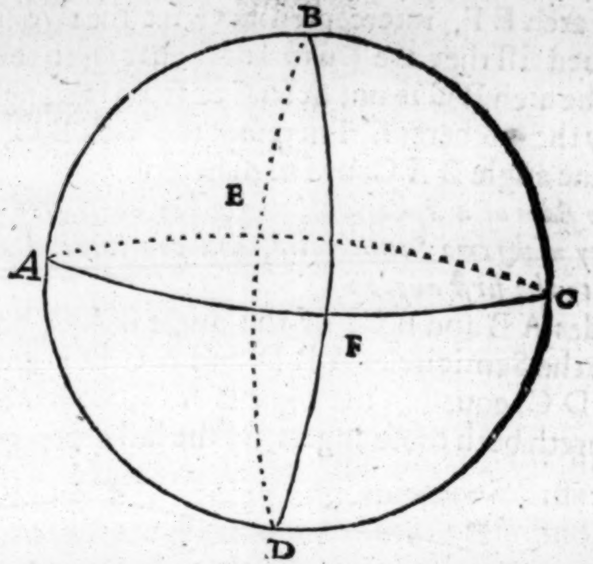
It followeth of Sphæricall.

55 *The sides of a Sphæricall Triangle, are the arches of great Circles, every one being lesse then a Semicircle.*

56 *A great circle of the Sphare, is that which divideth the whole Sphare into two Hemispheres, and so is every where distant from his Poles by a Quadrant of a great Circle.*

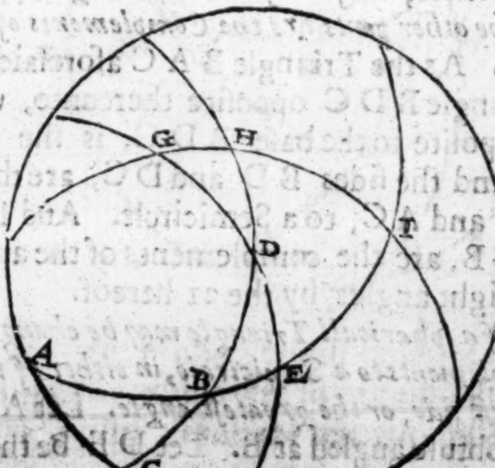
57 *If a great circle of the Sphare, passe by the Pole of another great Circle, they cut one another at right angles:* and *Contra.*

Let A E C, be a great Circle of the Spheare, whose Poles let be B and D, by which Poles B and D; let another great Circle passe being B E D; I say that the great Circle B E D, cutteth the great Circle A E C, at right angles, at the points E and F. For upon the Pole E or F, let also another great circle A B C D be described, it is manifest that the arches A B, B C, C D, and D A, shall be the measures of the angles E and F, by the 6. hereof. But the arches A B,



AB, BC, CD, DA, are Quadrants, by the last afore-going; Therefore the angles at E and F, are right angles by the 15 hereof, which was to be demonstrated.

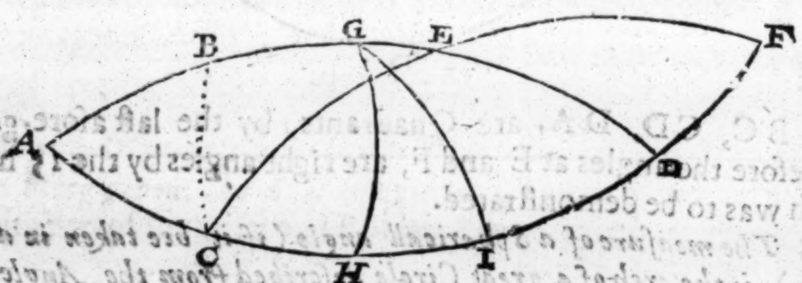
58 *The measure of a Sphericall angle (if it bee taken in a great circle) is the arch of a great Circle described from the Angle, and intercepted betwixt the two sides, being continued out till they are Quadrants*, by the 6. and 56 hereof.



As the measure of the Sphæricall angle B A C, is not the arch B C, but the arch E F, intercepted betwixt the two sides A B and B C, continued till they are *Quadrants*: that is, to the points E and F; because the arch B C is not described from the angle A, but the arch E F, by the 56 hereof. Therefore the arch B C, cannot be the measure of the angle B A C, by the 6. hereof.

59 *If the sides of a Sphæricall angle bee continued till they meet together, they make two Semicircles, and comprehend an angle equall and opposite to the first angle:*

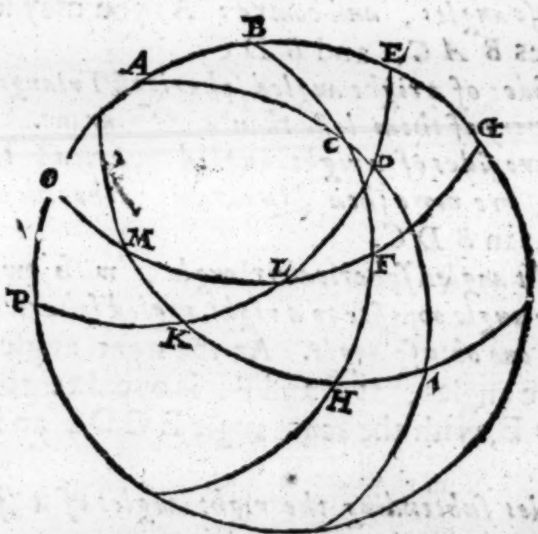
As the sides A B and B C, of the angle B A C, being continued to D, make the Semicircles A B D and A C D; and comprehend the angle B D C, equall to the angle B A C; because the same arch G H, measureth both those angles, by the last afore-going.



60 *Every Sphæricall Triangle, from every angle hath another Triangle opposite thereunto, whose Base and the angle opposite to the base, are the same: The other parts are the Complements of the parts of the former Triangle.* As the Triangle B A C aforesaid, from the angle A, hath the Triangle B D C opposite thereunto, whose base B C, and the angle opposite to the base B D C, is the same by the last afore-going: and the sides B D and D C, are the Complements of the sides A B and A C, to a Semicircle. And lastly, the angles D B C and D C B, are the complements of the angles A B C and B C A, to two right angles, by the 21 hereof:

61 *The sides of a sphæricall Triangle may be changed into angles, and contra: the complements to a Semicircle, in either of them? being taken for the greatest side, or the greatest angle.* Let A B C be a Sphæricall Triangle, obtuse angled at B. Let D E be the measure of the angle at A: Let F G be the measure of the acute angle at B, (which is the complem. of the obtuse angle B, being the greatest angle in the

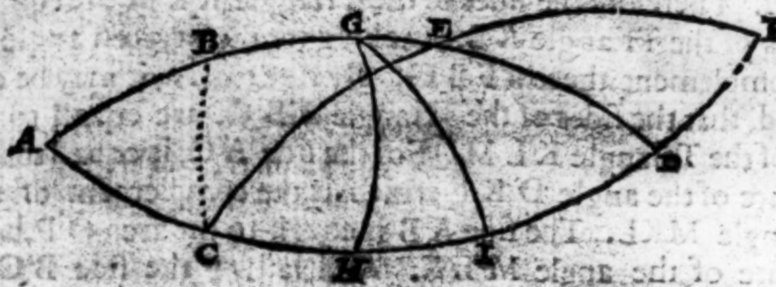
the given Triangle) and let H I, bee the measure of the angle, at C K L, is equall to the arch D E; because K D, and L E are Quadrants, and their common complement is L D. L M is equall to the arch F G; because L G and F M, are quadrants, and their common complement is L F. K M is equall to the arch H I, because K I, and M H are quadrants, and their common complement is K H. Therefore the sides of the Triangle K L M, are equall to the angles of the Triangle A B C, taking for the greatest angle A B C, the complement thereof F B G. By like reason, it may be demonstrated, that the sides of the Triangle A B C, are equall to the angles of the Triangle K L M. For the side A C, is equall to D I, the measure of the angle D K I, which is the complement of the obtuse angle MKL. The side A B is equall to the arch O P, being the measure of the angle M L K. And lastly, the side B C, is equall to the arch F H, being the measure of the angle L M K. For A D and C I, are Quadrants: so are A P and O B, B F and C H. And C D, A O, and C F, are the common complements of two of those arches.



Therefore the sides of a Sphæricall triangle, may be changed into angles, and *contra*: which was to be demonstrated.

62 ***A right angled Sphæricall triangle, hath one right angle or more then one.***

63 *One right angle with two acute angles*, *as* A B C, *or with two obtuse angles*, *as* B D C, *or with one obtuse and one acute angle*, *as* C D E. For I suppose the angles at *A* and *D*, to bee right angles.



64 *A right angled sphericall Triangle*, *with two acute angles hath from the right angle*, *a right angled Triangle*, *opposite thereunto*, *with two obtuse angles*, *and contra*: As you may see in the right angled Triangles *B A C*, and *B D C*.

65 *The sides of a right angled sphericall Triangle*, *with two acute angles*, *are every of them lesse then a Quadrant.* As in *A B C*.

66 *The two sides of a right angled sphericall triangle*, *with two obtuse angles*, *are more then Quadrants*; *the third side is lesse then a Quadrant.* As in *B D C*.

67 *A right angled sphericall triangle*, *with two acute angles*, *is from the acute angle opposite to a right angled sphericall triangle*, *with one acute and one obtuse angle.* As the right angled triangle E D F, with two acute angles, at E and F, is opposite to the right angled Triangle C D E, with the acute angle E C D, and the obtuse angle C E D.

68 *The sides subtending the right angles of a sphericall triangle*, *having divers right angles*, *are Quadrants.*

The reason is, for that; (as in the triangle *A G H.*) If the great Circles *A G* and *A H*, doe cut the great circle *G H*, at right angles in the points *G* and *H*. *A* is the pole of the great circle *G H*, by the 57. hereof. And *A G* and *A H*, are Quadrants by the 56: hereof.

hereof. But if the angle at *A*, be also a right angle, then *G H*, is also a Quadrant by the 58. and 15. hereof.

69 *A sphæricall triangle, having divers right angles, hath either three or two right angles: And so of the sides, hath three or two Quadrants.* As if you put the angle at *A*, for a right angle, the sphæricall triangle *A G H*, shall have three right angles at *A*, *G*, and *H*; and therefore the three sides also, *A G*, *G H*, and *A H*, shall bee Quadrants.

But if you put the angle at *A*, for an acute angle, then the sphæricall triangle *A G H*, shall have two right angles at *G* and *H*, and thereupon the two sides also, *A G* and *A H*, shall be quadrants.

70 *If the third angle of a sphæricall triangle, having two right angles, be acute, the third side is lesse then a quadrant. But if obtuse, then the third side is more then a Quadrant.* As in the sphæricall triangle *H G I*, acute angled at *G*, the third side *H I*, is lesse then a quadrant. In the sphæricall triangle *A G I*, obtuse angled at *G*, the thir side *A I*, is more then a Quadrant.

The former Diagram sheweth the Demonstration hereof.

71 *An oblique sphæricall triangle, consisteth simply of acute angles, or obtuse angles, or of both of them mixed together.*

72 *A sphæricall triangle, with two obtuse angles, and one acute angle, is opposite to a sphæricall triangle, simply acute angled. And contr:* As if the angles, at A and D, be supposed acute, then the triangle B D C, with two obtuse angles, at *B* and *C*, and one acute angle at *D*, is opposite to the simply acute angled triangle, A B C.

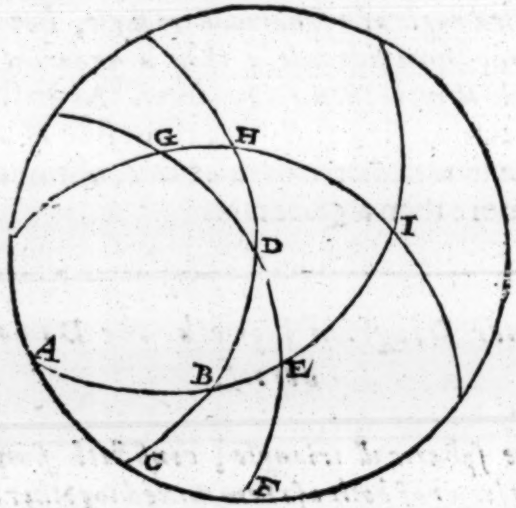
73 *A sphæricall triangle, with two acute angles, and one obtuse angle, is opposite to a triangle Sphæricall, simply obtuse angled: and contra:* As if the angles, at A and D, be supposed obtuse, then the triangle A B C, with two acute angles, at B and C, and one obtuse angle, at A, is opposite to the simply obtuse angled Triangle B D C.

74 *The three Angles of every sphæricall Triangle, are more than two right angles.*

In Sphæricall Triangles, having more right or obtuse Angles then one; whether they be simple or compound, the thing is manifest of it selfe.

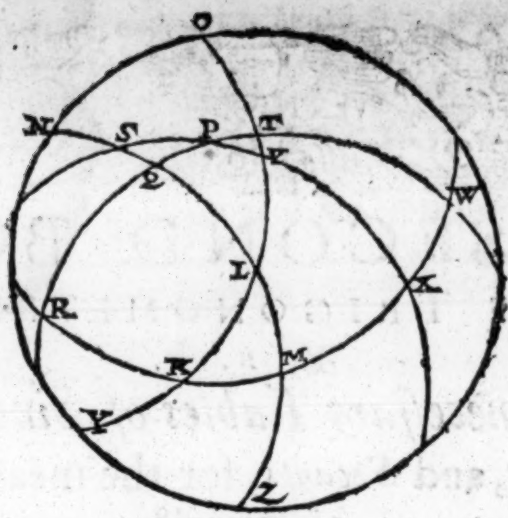
In Sphæricall triangles of two or three acute angles, it may bee thus demonstrated.

In the Sphæricall triangle A B C of two acute angles, right angled at C, and acute angled at A and B, the measure of the acute angle B A C, is the arch E F, and the measure of the acute angle A B C or D B E, is not the arch D E, but H I, by the 58 hereof.



But the arches E F and D E, are equall to a *Quadrant*. Therefore the arches F E and H I, added together, are more then a quadrant. And consequently, the angles answering to these arches; to wit, the angles B A C and A B C, joyntly together, are more then a *Quadrant*; that is, greater then a right Angle. But the angle A C B, is a right angle by the *Prob*: Therefore in the Sphæricall triangle A B C, of two acute angles, the three angles are more then two right angles.

In the Sphæricall triangle K L M, meerly acute angled: The measure of the acute angle at L, is the arch N O, the measure of the acute angle at K, is the arch V X; the measure of the acute angle at M, is the arch Q R.



But these three Arches, N O, V X, and Q R, added together, are more then two *Quadrants*. For P V, and P Q, (being the Complements of the two arches *Q R*, and V X,) added together, are lesse then the arch N O, by the Pro: Therefore the arch N O, being the measure of the third Angle, is more then the complements of the other two angles added together. And consequently, also the third angle is greater then the Complements of the other two angles. And therefore in sphæricall Triangles, meerely acute angled, the three angles are more then two right angles: A more subtill demonstration see in *Regiomont*.49. P. 3.

The end of the first Booke.

THE SECOND BOOKE OF TRIGONOMETRIA.

By *B. P.*

Of the necessary Tables of Sines, Tangents, and *Secants*, for the measuring of Triangles.

1.

S*O are Triangles: The measure of Triangles, is the finding out in Triangles the unknowne sides or Angles; by three knowne, whether Angles, or sides, or both.* It is also called the resolving of Triangles, or the calculating of Triangles.

There are *Area's* in Triangles, besides their Angles and Sides, but the measure of them is not proper to Triangles; for we measure the *Area* of any other figures whatsoever aswell as of them: Neither commeth it first from Triangles, but is derived from Quadrangles to Triangles. And therefore appertaineth not to this place.

2 *The dimension of Triangles, is performed by the golden Rule of* Arithmetick *: which teacheth of foure Numbers proportionall one to another, any three of them being given, to find out the fourth.*

3 *Therefore for the measuring of Triangles, there must be certaine proportions of all the parts of a Triangle one to another, and those proportions explained in Numbers.*

4 *The proportions of all the parts of a Triangle one to another cannot be certaine, unlesse every crooked Line in triangles (as in all Triangles the measure of the angles are, and in Sphericall triangles also the Sides) bee reduced to right Lines. For of a crooked line, to a crooked line, or to a right Line was never yet found any proportion, nor perhaps shall ever bee.*

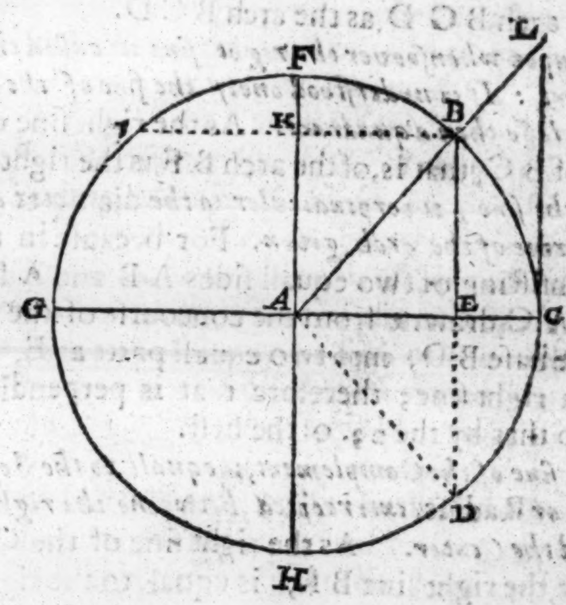
5 *Crooked lines are reduced to right lines, by the definition of quantitie, which right lines applyed to a Circle have, in respect of the Radius.*

6 *Right lines applyed to a Circle are Subtenses,* Sines, Tangents, *and* Secants.

7 *A Subtense is a right line, inscribed in a Circle, dividing the whole Circle into two Segments, and in like manner subtending both the Segments.*

8 *A Subtense is either the greatest, or not the greatest.*

9 *The greatest Subtense, is that that divideth the whole Circle into two equall Segments.* And so it subtendeth both the Semicircles as the right line G C, is commonly called a *Diameter*.



10 *A Subtense not the greatest, is that that divideth the whole Circle into two unequall Segments: And so on the one side Subtendeth an arch lesse then a Semicircle, and on the other side subtendeth an Arch more then a Semicircle.* As the right line I B, which on the one side subtendeth the arch I F B, lesse then a Semicircle: and on the other side subtendeth the arch I H B, greater then a Semicircle.

11 A *Sine is either right, or versed.*

12 A *right Sine is the one halfe of the subtense of the double arch,*

As the right sine of the arches B C, or B G, is the right line B E, being the ½. of the subtense of the double arches of B C, or B G, that is, the ½. of the right line B E D, which subtendeth the arches B C D, or B G D. So the right sine of the arches B F, or B H, is the right line B K, that is the ½. of the right line B K I, which subtendeth the double arches of B F, or B H, to wit, the arches B F I, or B H I.

Consectaries.

1 *Therefore the right sine of an arch, lesse or more then a Quadrant, and lesse then a Semicircle, is one and the same.* As the sine of the arches B C, and B G, is the same right line B E, for that it is the ½. of the right line B E D, which subtendeth aswell the arch B G D, as the arch B C D.

2 *And thereupon whensoever the right sine is called the sine of the Complement: It is understood onely the sine of the Complement of an arch lesse then a quadrant.* As the right sine of the Complement of B C, that is, of the arch B F, is the right line B K,

3 *Every right sine, is perpendiculer to the* diameter *drawne from the one tearme of the arch given.* For because in the triangle A B D, consisting of two equall sides A B, and A D, the semidiamiter A C drawne from the concourse of the equall sides, cutteth the base B D, into two equall parts at E, by the definition of a right sine; therefore that is perpendiculer to this and this to that, by the 23. of the first.

4 *The right sine of the Complement, is equall to the Segment of the* Diameter *or* Radius, *intercepted betweene the right sine of the Arch, and the Center.* As the right sine of the Complement B F, to wit, the right line B K, is equall to the right line E A, by the 39. of the first.

13 *The versed sine, is the segment of the* Diameter *intercepted betwixt the right sine, and the Circumference.* As the versed sine of the arch B C, is the segment of the *Diameter* E C, the versed sine of the arch B G, is the segment of the *diameter* G E.

14 *Therefore of versed sines, some are greater and some lesse.*

15 *A greater versed sine, is the versed sine of an arch, greater then a Quadrant.* As C E, is the versed sine of the arch G F B, being greater then a quadrant.

16 *A lesser versed sine, is the versed sine of an arch lesse then a*

Qua-

Quadrant. As E C, is the versed sine of the arch B C, being lesse then a quadrant.

17 *A* Tangent, *is a right line drawne (from the* Secant) *by one end of the arch, perpendicular on the extremity of the* Diameter, *passing by the other end of the earth.* As L C, is the *Tangent* of the arch B C.

18 A Secant, *is a right line drawne by the one end of the arch, to the toppe of the* Tangent. As the *Secant* of the arch B C, is the right line A L.

19 *The definition of the quantity which right lines have applyed to a Circle, is the making of the tables of* Sines, Tangents *and* Secants; that is to say, of right sines, and not of versed; For the versed-sines are found by the right sines without any labour. *For the lesser versed sine, with the right sinne of the Complement, is equall to the Radius.* As the lesser versed sine E C, with the right sine of the Complement A E, is equall to the *Radius* A C. Therefore if you subtract the right sine of the Complement A E, from the *Radius* A C, there resteth the versed sine E C. *But the greater versed sine is equall to the* Radius *added to the right sine of the excesse of the arch, more then a Quadrant*; As the greater versed sine G F, is equall to the Radius G A, joyned with the sine of the excesse A E. Therefore if you adde the right sine of the excesse A E, to the Radius G A, you shall have the versed sine of the arch G F B, and therefore there is no need of the Table of versed sines. In stead of the subtenses, the right sines may be used: for the right sines are the ½. of the subtenses; Therefore if you take the greatest sine for the greatest subtense, you may also take the lesse sine for the lesser subtense: For the same reason is, of the halfe to the halfe, as is of the whole to the whole: As what proportion 10. hath to 6. the same proportion 5 hath to 3.

20 *The tables of* Sines, Tangents, *and* Secants, *are commonly called the Canon of Triangles.* Rhæticus *calleth it the Canon of the doctrine of Triangles.* Vieta *the Mathematicall Canon.*

21 *The tables of* Sines, Tangents, *and* Secants, *are extended no farther then to a Quadrant.* For the right sines of arches more or lesse then a quadrant, are the same by the 12. hereof. *And there can be no* Tangents *and* Secants *of arches greater then a quadrant*, by the 17 and 18 hereof.

22 *The tables of* Sines, Tangents, *and* Secants, *are commonly made*

made to Minutes : Rhæticus *made them to Tenths of Seconds :* I in the beginning and end of a Quadrant, have calculated them to seconds, one, two or ten, as necessity required : In the rest I have been contented with the setting downe the Minutes.

23 *First of all, for the making of the Tables of* Sines, Tangents, *and* Secants : *The* Radius *is to be taken of a certaine number of parts.*

24 *Of what parts soever the* Radius *be taken, the* Sines, Tangents *and* Secants, *for the most part are all of them irrationall to it, that is, inexplicable in any true whole Numbers, or Fractions, precisely* by the 51 of the first. *And therefore the Tables of* Sines, Tangents, *and* Secants, *cannot bee exactly made by any Meanes : yet such may and ought to bee made, wherein no Number is different from the truth, by an intiger of those Parts, whereof the* Radius *is taken.* As if the Radius bee taken of 10000000. no Number of those Tables ought to bee different from the truth by 1. of 10000000.

25 *That you attaine this exactnesse, eyther you must use the Fractions, or else you must take the Radius, for the making of the Tables much greater then the true Radius.*

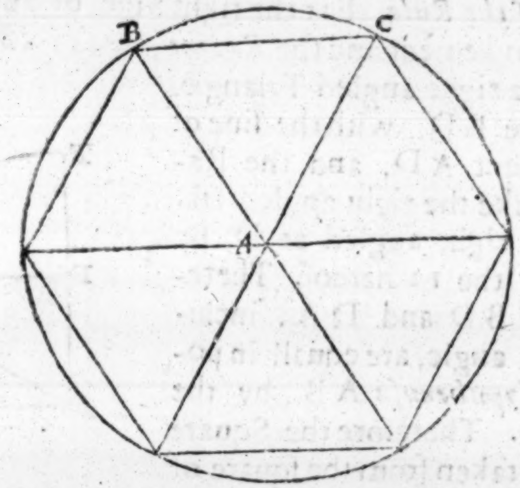
26 *But to worke with whole Numbers and Fractions in the Calculation is very tedious : Besides, here no Fractions almost are exquisitely true : Therefore the Radius for the making of these Tables is to bee taken so much the more, as there may be no errour in so many of the figures towards the left hand, as you will have placed in the Tables : And as for the Numbers superfluous, they are to bee cut off from the right hand towards the left, after the ending of the supputation.*

So did *Regiomontanus,* when hee would calculate the table of Sines to the Radius of 6000000. hee tooke the Radius of 60000000000. And after the supputation ended, hee cut off from every Sine so found, from the right hand towards the left, 4 Figures. So *Rhaticus,* when hee would calculate a table of Sines, to the Radius of 10000000000, hee tooke for the Radius 1000000000000000. And after the supputation ended, he cut off from every Sine found from the right hand towards the left, 5 Figures. But I to find out the Numbers in the beginning of the Table, tooke the Radius of 10000000000000000000000000. But in the *Canon* it selfe, have taken the Radius of divers Numbers for necessity sake : As hereafter in his place shall be declared.

27 *In*

27 *In the beginning you shall find out the right Sines of all the arches lesse then a Quadrant, in the same parts as the* Radius *was taken of whatsoever bignesse : Then out of those right Sines you shall find the* Tangents *and* Secants.

28 *The right Sines (in the making of the Tables) are either primary or secondarie. The primary Sines are those by which the rest are found.*



29 *Now I make the totall Sine, or the Radius the first primarie sine, which is equall to the side of the Six-angled figure inscribed in a Circle, that is to the subtense of 60 Degrees.* Which is thus demonstrated. Let B C bee the side of a six-angled figure, inscribed in a Circle : Then because the arch B C, is 60 parts by the *Pre*: therefore also the angle B A C, is 60 parts by the 6th of the first : And thereupon the angles A B C, and A C B together, are 120 parts by the 49 of the first ; but the angles A B C and A C B, are equall, by the 5 of the first ; for the sides A B and A C, opposite unto them, are equall, that is, two *Radii* ; Therefore either of the angles is 60 parts : but the angle B A C, was also 60 parts ; Therefore the triangle A B C, is equiangled by the 5 of the first ; but the sides A B and A C, are *Radii* by the worke ; and therefore the side B C, is Radius also. Therefore the totall Sine or the Radius, is equall to the side of a Six-angled figure, inscribed in a Circle, which was to be demonstrated.

30 *Out*

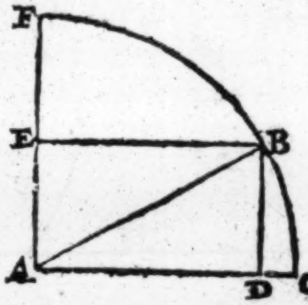
30 *Out of the totall Sine, I deduce all the other sines, by the 9 Problems following.*

The first Problem.

The right sine of an Arch lesse then a Quadrant being given, to find the sine of the Complement.

The Rule. *Subtract the square of the sine given, from the square of the* Radius : *The square root of the Remainder, is the sine of the Complement.*

The reason of the Rule. For the right Sine of any Arch with the sine of the Complement and the *Radius*, make in the meeting of the two sines a right angled Triangle, as the right sine B D, with the sine of the Complement A D, and the Radius A B, make the right angled triangle B D A, right angled at D, by the 3 *Con:* of the 12 hereof. Therefore the sides B D and D A, including the right angle, are equall in power to the *Hypothenusa* A B, by the 30 of the first. Therefore the Square of B D, being taken from the square of A B, the Remainder is the square of A D, whose square Root is A D or E B, the sine of the Complement; that is, of the arch F B.

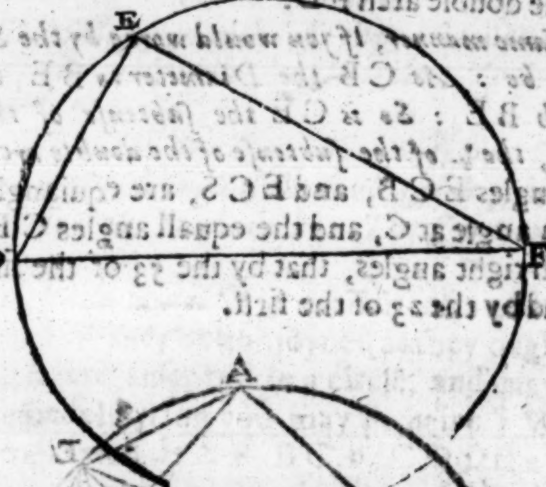


Example. Let the Radius A B, bee 10000000, the sine B D; that is the sine of the arch of 30 deg. 5000000. The square of the Radius A B, is 100000000000000. The square of the sine B D, is 25000000000000. the which if you subtract from the the square 100000000000000. The rest shall bee the square 75000000000000, whose square Root shall bee 8660254. the sine A D or E B, serving for the arch F B, 60 deg.

After the same manner. The subtense of an arch lesse then a Semicircle being given, you may find the subtense of the Complement to the Semicircle.

For as the sine of any Arch, with the sine of the Complement and the Radius doe make a right angled triangle, by the third *Cons:* of the 12 hereof. So the subtense of any arch with the subtense of the Complement to a Semicircle and the Diameter, make a right angled

angled Triangle, by the first *Con:* of the 53 of the first ; therefore if you take the square of the Subtense given, from the square of the Diameter, the Remainder shall be the square of the subtense of the Complement : As in the Diagram propounded. If you take the square of the subtense D E, from the square of the Diameter D F, the Remainder shall be the square of the subtense E F.

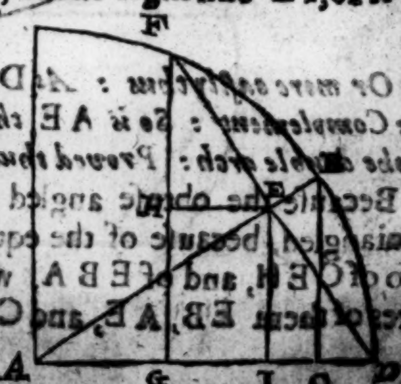


The second Problem.

31 *The right Sine of an Arch being given, with the sine of the Complement, to find the Sine of the double arch.*

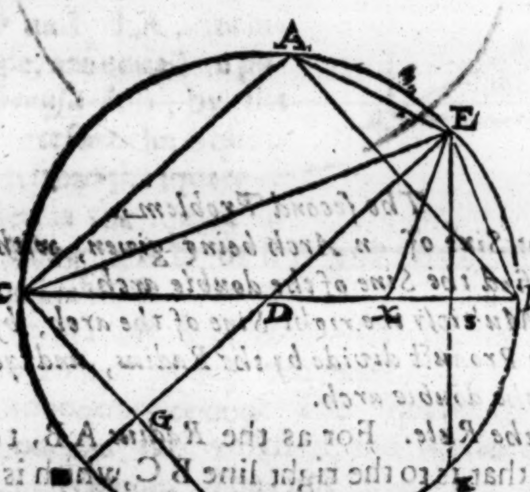
The Rule ; *Multiply the right Sine of the arch, by the sine of the Complement, the Product divide by the Radius, and you shall have the ½ of the Sine of the double arch.*

The reason of the Rule. For as the *Radius* A B, to the sine of the arch given B D, that is to the right line B C, which is equall to D E. So is the sine of the complement A E, to the right line E I, or H G, whose double F G, is the Sine of the arch F D. For in the Triangle F G D the right line H E, being Paralell to the base G D, cutteth the sides F G, and F D, proportionally, by the 45 of the first. But it cutteth the side F D, into two equall parts, in E. And therefore it cutteth the side F G, in H, into two equall parts also. *Example.*



Example. Let the sine of the arch B D, 35 degr. be given, the right line E D, or B G, 5735764, together with A C, or A E, the sine of the Complement 8191520. And let the sine of the double arch, to wit, the sine F G, bee demanded. I say: As A B, 10000000. to B G, 5735764. So is A E, 8191520. to E I, or H G, 4698462. which doubled, is 9396924. F G, the sine of 70 degr. being the double arch F D.

After the same manner, if you would worke by the Subtenses; The proportion shall be: As C B *the Diameter to* B E, *the subtense of the simple arch* B E : *So is* C E *the subtense of the Complement* C A E *to* E S, *the* ½. *of the subtense of the double arch* E B K. Because the Triangles E C B, and E C S, are equiangled, because of their common angle at C, and the equall angles C E B, and C S E, which are both right angles, that by the 53 of the first, and this by the worke, and by the 23 of the first.



Or more easily thus: As D E *the Radius, to* E C, *the subtense of the Complement: So is* A E *the subtense given to* A B, *the subtense of the double arch: Proved thus.*

Because the obtuse angled Triangles, C D E and E A B, are equiangled, because of the equall angles E C B and E A B; and also of C E H, and of E B A, which are equall; because the measures of them E B, A E, and C H, are equall by the *Pro-*

Example of this last manner. Let E B, the subtense of 50 degr. given, bee 8452366. together with the subtense of the Complement C E, 18126156. And let A B, the subtense of the double arch be sought for. I say,

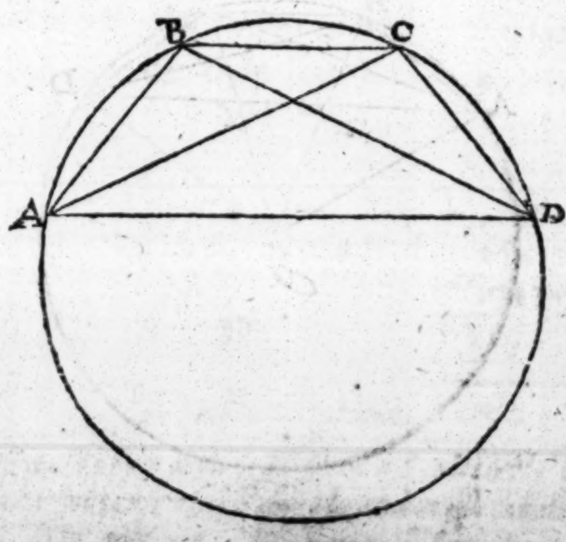
As D E, 10000000. to E C, 18126156. So is E B, 8452366. to A B, 15320890.

The third Problem.

32 *The Subtense of an arch lesse then a Semicircle, being given, together with the subtense of the double arch: To find the subtense of the triple arch.*

The Rule. *Take the Square of the subtense of the simple arch from the square of the subtense of the double arch; divide the Remainer by the subtense of the simple arch: The Quotient shall be the subtense of the triple arch.*

The reason of the Rule. For the subtenses of the simple, double and triple Arches, if they be conjoyned as they ought, doe make a Quadrilaterall figure, inscribed in a circle, and cut with Diagonals. As in the Scheme following you may perceive: Wherein the subtense of the simple arch, is A B, B C, or C D; the subtense of the double arch, is A C, or B D. The subtense of the triple arch, is A D. But in such a figure, the right angled figure, made of the Diagonals, is equall to the right angled figures made of the sides opposite one to another, by the 54 of the first.



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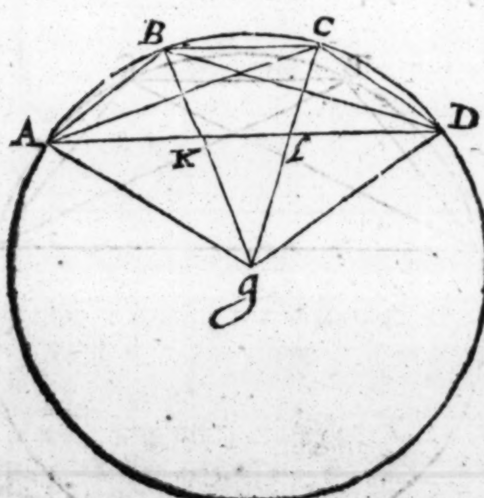
Therefore if I subtract the right angled Figure, made of the sides A B and C D; that is, the Square of the simple arch, from the right angled figure made of the Diagonals, that is, from the square of the double arch A C, the subtense of the double arch; there shall rest the right angled figure, made of the sides B C and A D, which divided by the side B C, the quotient will be the side A D, by the 40 of the first; which was to be demonstrated.

Example. Let A B or B C, the subtense of 10 deg. be 1743115. together with the subtense of 20 degr. A C, 3472964 be given; and let the subtense of 30 degr. A D, be sought for.

The square of the subtense A C, is	12061478945296
The square of the subtense A B, is	3038449903225
Which subtracted, resteth the right angled figure, made of B C and A D;	9023029042071
Which divided by the subtense B C, is	1743115
The Quotient is the subtense A D,	5176381

Or more easily without the Subtense of the double Arch given? Subtract the square of the subtense given, divided by the Radius, *from the* Radius: *The rest multiplied by the subtense given, and divided by the* Radius; *adde to the double of the subtense given: And you shall have the subtense of the triple Arch.*



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The reason of the Rule, appeareth by the Scheme adioyned, wherein first the Triangles A G B, and B A K, are equiangled because of their common angle, A B K, or A B G, and their equall angles *A G B*, and *B A K*, which are equall by the 51. of the first; for that the arch B C D, which lyeth against the angle *B A K*, or B A D, beeing in the circumference, is double to the arch A B, which is opposite to the angle in the center A G K.

Therefore as A G, to A B. So is A B, to B K, which subtracted from BG, resteth K G. Then the triangles G B C, and GK L, are also equiangled, because the bases BC, and KL, are paralels by the 38. of the first: Therefore as B G, to B C, so is G K, to K L. And lastly, the Triangle BAK, is equiangled at the base, for it is like to the Triangle AGB, which is equiangled at the base, as before was demonstrated: Then because the Triangle B A K, is equiangled at the base, therefore the two sides are equall by the 62 of the first, and consequently the two sides A B, and A K, are equall. But the segments AK, and L D, are also equall, by the worke. Therefore if I adde A K, and L D, to K L. It is all one as if I should adde the right line A B, twice to the right line K L.

Example. Let the same subtense A B, be given as before to wit, the subtense of 10. deg. 1743115, And let AD. the subtense of the triple arch be sought for.

The square of the subtense A B given, is	303844	9903225
The right line B K, is ————————	303845	
Which subtracted from the *Radius* given,	10000000	
The remainer shall be the right line K G,	9696155	
which multiplied by A B, the right line given,	1743115	
Produceth the right angled figure, ———	1690151	3222825
which divided by the *Radius* ỳ quotient is KL,	1690151	
To which the right line A B, twice added,	1743115	
	1743115	
Maketh the right line A D, ———	5176381	

The fourth Probleme.

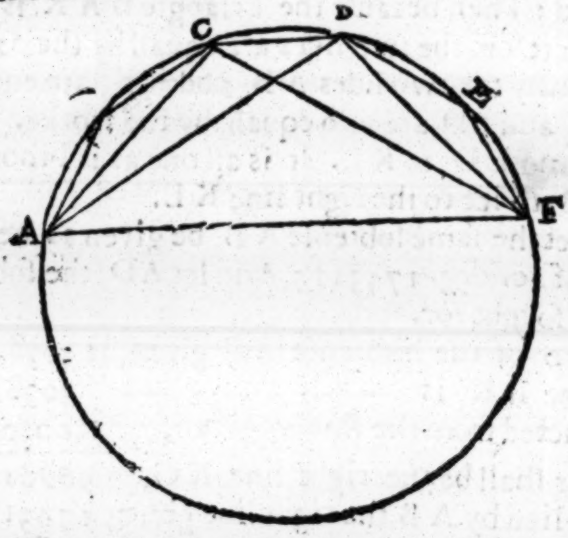
33 *The subtense of an arch lesse then a Semicircle being given, together with the subtense of the double and triple arch; to find the subtense of the arch quintuple, or of an arch five times as much.*

The Rule. *Take the square of the subtense of the double arch, from the square of the subtense of the triple arch, the remainder divided by the subtense given, shall be the subtense of the quintuple arch.*

The Reason. Is the same which was in the first solution of the third Problem; Forasmuch as the subtenses of the simple, double, triple, and quintuple arches, truly conjoyned one with another, doe make a quadrilaterall figure, intersected with two Diagonals, and so is to be applyed to the 54 of the first, &c.

Example, Let C D be given, the subtense of 2. degrees 349048 and let A F, the subtense of 10. degrees be sought for.

First, the subtense of the double arch is to be found: that is, the subtense of the arch A C, 4. degr. by the second Problem. And the subtense of the triple arch, that is, the subtense of the arch A D, 6. degr. by the third Problem.



The subtense A C, shall be 697990. almost.
The subtense A D, shall be 1046719.
Then square those subtenses and they shall be as followeth.
The square of the subtense of the triple arch AD, 1095620664961
The square of the subtense of the double arch AC, 487190040100

Which subtracted from the square AD, there remaines the right angled figure, made of A D, and C D. ———— } 608430624861.

Which divided by the side C D, ———— 349048
The Quotient is AF, ———— 1743114

Note.

Note. By the same reason if need be, you may find the subtenses of the arches, 7 times, 9 times, 11 times, &c. as much, as the subtense of the arch given. For the Square of the subtense of the triple arch subtracted from the Square of the subtense of the quadruple arch, leaveth a Number which divided by the subtense of the simple arch, giveth in the quotient the subtense of an arch 7 times as much as the simple arch. So the square of the subtense of the quadruple arch, subtracted from the square of the subtense of the quintuple arch, leaveth a Number, which diuided by the subtense of the simple arch, bringeth out in the quotient, the subtense of an arch, nine times as much as the simple arch. And so forward infinitly.

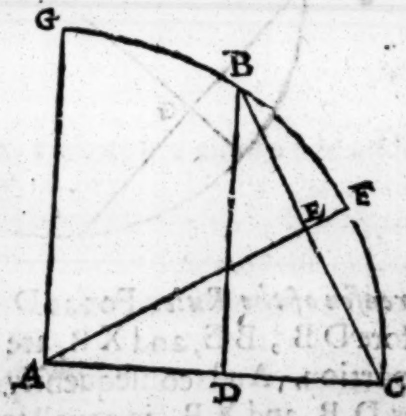
The Fift Problem.

34 *The sine of an Arch being given, together with the sine of the Complement; to find the sine of halfe the arch giuen.*

The Rule. *Adde the square of the right sine of the Arch, giuen to the square of the versed sine of the same Arch,* (which versed Sine you shall find, by subtracting the sine of the Complement, from the *Radius*(*The square Root of the summe of these two squares, shall bee the subtense of the Arch given, whose halfe shall be the sine of halfe that arch.*

The reason of the Rule. For the right sine, and the versed sine are equall in power, to the subtense of their arch.

As in the Scheme adjoyned B D, the right sine of the arch B C, and D C, the versed sine of the same arch, are equall in power, to the subtense of that arch B C, by the 50 of the first; the ½. of which subtense, being E C, is the sine of ½. the arch, being F C.



Example. Let the sine of the arch B C, 30 deg. be the right sine B D, 5000000, and the versed sine of that arch, be D C, 1339746

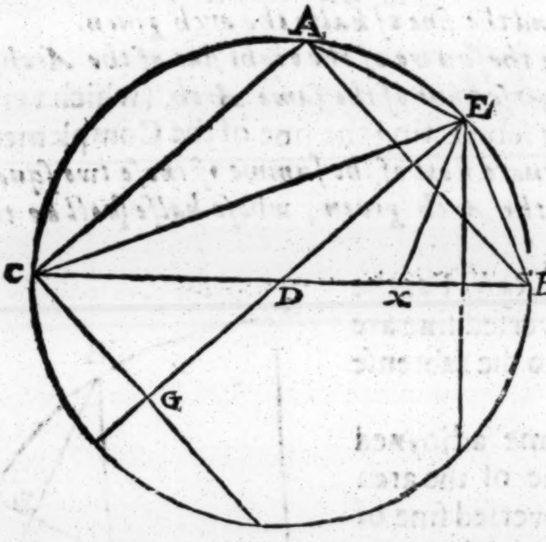
The q. of the right sine B D, shall be 25000000000000.
The q. of the versed sine D C, shall be 1794919344516.

The totall of these two squares, shall be 267949193445 16. whose q. l. 5176380. shall be the Subtense of the arch given BC, 30. deg. The ½. of which subtense, that is to say, the right line, EC, 2588190. shall be the sine of ½. that arch being F C, 15. deg.

Otherwise, by the *Subtenses.*

The Rule. *Take the subtense of the Complement, from the Diameter, the Remainder multiplyed in the* Radius, *shall be the square of the subtense of halfe tha Arch.* As for Example.

Take the subtense of the Complement A C, (equall to C X,) being the subtense of the complement of the arch given AB, from the Diameter C B. The Remainder XB, multiplyed in the *Radius* D B, shall be equall to the square of the right line E B, being the subtense of the halfe arch E B.



The reason of the Rule. For as D B, to E B; so is E B, to X B, Therefore D B, B E, and X B, are three right lines, in continuall proportion. And consequently, the oblong made of the extreames D B, and X B, is equall to the q. of the Meane B E, by the 43. of the first. And for this cause it is, that as D B to B E; so is B E, to X B, for that the Triangles D E B, and B E X, are equiangled, because of their common angle D B E, and their equall angles E X B, and D E B, which are equall one to another; that is to say, for that they are equall to a third, to wit, their common angle D B E. And therefore, because the Triangles

angles, DEB, and XEB, are equicrurall, and they are equiangled at the base, by the 26. of the 1. The triangle DEB, is equicrurall, because either of the sides, D E, and D B, is the *Radius*: The Triangle X E B, is equicrurall because the right line X E, is equall to the right line A E, and therefore also to the right line EB. For the right lines A E, and E B, are equall by the worke. And the right line E X, is equall to A E, because they subtend the equall angles, ACB, and ECX, in the termes of their equall sides, For the right line C X, is equall to the right line C A, by the Pro: but the right line, C E, is common to both the Triangles, to wit, to the Triangles A C E, and E C X.

Now because the Triangles, D EB, and XEB, are equiangled, therefore as D B, to BE, so is BE, to BX, which was to bee demonstrated.

Example:

Let the subtense of the arch, A B, 60. deg. be given,	10000000
together, with the subtense of the Complement, A C.	17320508
From the diameter C B, ———————————	20000000
I subtract the subtense of the cōplement, A C, or C X,	17320508
The remainder shall be X B, ———————————	2679492

which multiplyed by the *Radius*, D B, that is adding 7. ciphers, after this manner, 26794920000000. shall be the q. of the subtense of the halfe arch, E B, whose q. l. is 5176381, the said subtense E B.

But then in these operations, ciphers are also to bee added in the beginning, if the calculation so require it, that the pricke of the Number to bee extracted, (whether the same bee square as heere; or cubicke, or solid as it will be in some of the examples following, (may duly bee noted. For the Numbers from the right hand, if a great *Radius* bee taken, are not alwayes to bee written downe. In which case the noting of the Radicall pricke should bee vncertaine, if ciphers were not added in the beginning. But this adding of Ciphers in the beginning, hath an other vse, for it sheweth that all these subtenses are lesse then the *Radius*, and as it were certaine parts of the *Radius*, which parts are commonly thus written, $\frac{5176381}{10000000}$. But much more briefe and necessary for the worke, is this writing of it, 5176381. For

 those

those numbers are altogether of the same value, as these two numbers, 09 and $\frac{9}{10}$ are.

Yet otherwise by the subtenses and by Algeber, of the invention of Iustius Birgius.

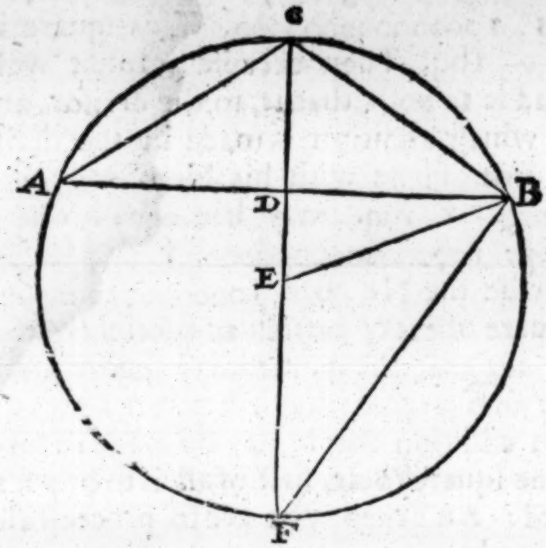
He that knoweth not *Algeber*, let him leave the Algebraicall worke here, and throughout the whole booke, for these examples are not put of necessity, but onely of curiosity.

The rule: *Divide the square of the subtense, given by* 4q — 1bq. *the quotient shall be the* q. *of the subtense, of halfe the arch.*

The reason of the rule. For the square of the subtense, of any arch whatsoever shall bee equall to 4. squares, lesse by one biquadrat of the subtense of halfe the arch. Which is thus demonstrated.

Let the right line A B, bee given for the subtense of the arch *A C B*, And let the subtense of ½ that arch, to wit, the right line *A C*, or C B, bee demanded. Let the diameter F C, bee 2 that the *Radius* may bee made 1, as it is put in the Table of sines, although there many ciphers bee added to 1. which heere there is no need of. Then let the subtense *A C*, or C B being demanded bee put algebraically for one roote or side; and so is *C B*, 1. Roote: therefore the square of C B, shall bee 1q. for 1.l. multiplyed by 1. l. giveth 1q. If you take this square from the square of the diameter 2. to wit. from 4. there shall rest, 4. — 1q. which is the square of the right line F B, by the 50. of the 1. Because the Triangle F C B, is right angled at *B*, by the 1. consect. of the 53. of the first: Therefore the right line F B, is the roote of the Square, of 4 — 1q. which Root may be thus noted. 4. — 1q. or so: l. 4. — 1q. as every one hath accustomed himselfe. Let also the *Radius* EB, be drawn to make the Triangle, E F B. Now the triangles, E F B, and A C B, are equiangled; because of their equall angles C F B, and A C B, which are equall, because of their equall (or rather the same) measure, which is the arch C B.

But the angle C B A, is equall to the angle C A B, and the angle E B F, is equall to the angle C F B, or EFB, by the worke: therefore



therefore also the third angle A C B, is equall to the third angle F E B. by the 4 consect: of the 49 of the 1. Then, because the triangles E F B, and A C B, are equiangled, therefore is it, as E F, 1. to F B, l. 4. — 1 q. so is A C, 1. l. to A B. In which worke, that you may multiply the second tearme, by the third, because the second tearme, is a surd number, make the third tearme also a surd number, by multiplying 1. l. by it selfe to make 1q. which being done, the right line A C, shall bee l. 1q. Then multiply l4. — 1q. by l. 1q. after this manner.

The Number to be multiplyed l. 4 — 1q.
The Number multiplying — l. 1q.
The Product ——————— l. 4q. — 1bq.

Therefore the subtense *A C*, is equall to the Roote of foure squares, lesse by one biquadrat of the roote, assumed *A C*: And consequently the square of the subtense *A B*, is equall to foure squares lesse by one biquadrat of the assumed roote, or of the subtense of halfe the arch, *A C*, or *C B*; And so if you divide the square of the subtense, *A B*, by 4 q. — the quotient shall be the square of the subtense of the halfe arch *A C*, or *C B*, which was to be demonstrated,

But how the square of any subtense given, may bee divided by 4q. — 1bq. I will make manifest in the example following.

Example. Let the ſubtenſe given, of 60. deg. A B, be 10000000 whoſe ſquare is, 100000000000000. This ſquare is to be divided by, 4q — 1bq. Then becauſe I cannot well divide by 4q — 1bq. I adde to both, that is, to the diviſor, and to the dividend 1bq. (which addition is made in the diviſor, by taking away the ſine of leſſe with his Number,) that 4q. and 100000000000000. + 1bq. may bee equall one to another. And then I divide 100000000000000. + 1bq. by 4q. or which is all one, I divide the Number 100000000000000 by 4. So as I adde the ſquare of every particular quotient, (which in truth is a biquadrat: for that the quotient is a ſquare) with his complement, to the number to be divided, before I move forward the diviſor. Which addition that it may be made in his due places, the prickes of the ſquare roote, firſt of all is to be put to the number to be divided: And then you are to proceed after the ſame manner as followeth.

The ſquare of the ſubtenſe, of 60. deg. A B, is 2000000000000000.

100000000000000	The ſquare of 2. is 4. B.
(4) ſay 4. in 1. (0	
4. in 10. (2	
B Ad 4 to a 100. it	
makes 104. Say twice 4. is 8. which ſubtract	The ſquare of 6. is 36. The Complement is 24.
reſts 2400.	The totall is 276. D.
(4) Say 4. in 24 (6	And that Complement is found by multiplying the Root 6. by the double of 2. the Root going before, that is by 4. For 4. times 6. is (24
D 276. added	
2676	
24 ſubtracted	
reſts 27600	
(4) — (7	
3689 added	
31289	
28 ſubtract.	
reſts 328900	Hence forward the Numbers, to be ſubtracted are not
(4) — (9	
48141 added	

rests 1704100
(4 (4
214335 ad.

rests 31843600
(4) (9
4820001 ad.

rests 66660100
Ad. (4) 5358981 (1

rests 3201908100
(4) (9
Ad. 482308461

rests 8421656100
(4 (2
Ad. 1071796764

rests 149345286400
(4) (4
Ad. 21435935376

rests 1078122177600
(4) (3
Ad. 1607695154449

rests 388916930490 0(1
Ad.(4)45358983848 61

rests 4250676897610 0(1
Ad.(4)535898384862 1

rests 786575282472100
(4) (2
Ad. 1071796769724 44

93754989444544.

not put downe, even as they are not used to be put downe, in vulgar division.

2 should follow.

The

The processe of the particular Biquadrants, from whence *the whole Biquadrate is leasurely composed.*

```
    00 ( 2
         2
A.  4 (6
      46
B.   276
     676 (7
       527
C.    3689
   71289 (9
      5349
    048141
   7177041 (4
       53584
      214336
   717918436 (9
       535889
      4823001
   71796666601 (1
        5358981
   7179672019081 (9
         53589829
        482308461
   717967684216561 (2
          535898382
         1071796764
   71796769493452864 (4
          5358983844
         21435935376
   7179676970781221776 (3
            53589838483
           160769515449
```

A. The square of the Roote. 2.

B. The square of the Roote 6. with his Complement. The square of the Roote 6. is 36. The complement made by the multiplication of double the roote precedent, and this Roote 6. is 24. These after their due order (as is afore-shewed) added together make 276.

C. The square of the Roote 7. with his complement made by multiplying the double of 26. the Root afore-going in this Roote 7.

717967697238891693049 (1
            535898384861

71796769724425067689761 (1
            53589838484621

7179676972447865752824721 (2
            53589838486222
            107179676972444

717967697244893754959444544. the whole biquadrat.

The proof of the former Resolution, by changing it contrarily.

1 q:	26794919243112	
4 q.	107179676972448	
1 bq.	7179676972448	Subtracted.

Resteth 100000000000000, the square of the subtense of 60. degrees.

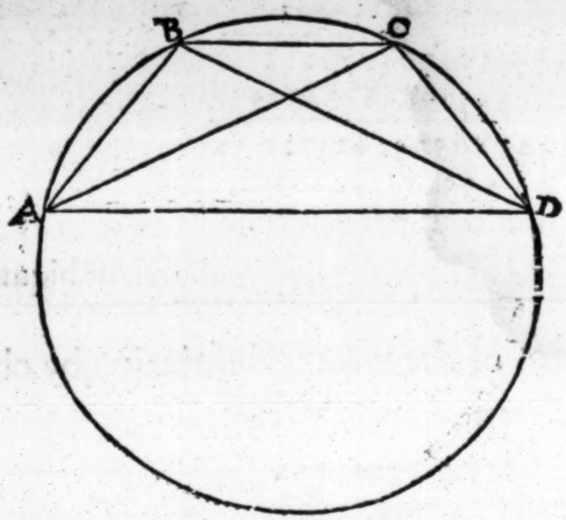
The Sixth Problem.

35. *The subtense of an arch being given, to find the subtense of the third part of that arch.*

The Rule. *Put the third part (of the subtense given) somewhat augmented, for the subtense demanded, and by that subtense, finde out the subtense given, according to the doctrine of the third Problem: Which if you find the same, you have that you sought for: But if otherwise, note the difference by more or lesse, and when you have repeated the same worke, by an other position of the subtense sought for, Againe, note the difference by more or lesse: Which being done, afterwards by the rule of false, you shall infallibly find the truth.*

The reason (why the third part, and somewhat more of the subtense given, may be put probably for the subtense sought for) is thus: because the subtense of the third part of the arch three times taken is greater of necessity then the subtense of the triple arch: As for example, the three right lines A B, B C, and C D, put together, are of necessity greater then the right line A D.

The



The reason of the rest of the worke untill you come to the rule of false, appeareth by the demonstration of the third Problem.

The reason of the rule of false, is knowne by Arithmetitians. *Example.* Let the subtense of 30. degrees be given AD, 5176381. almost. And let the subtense of $\frac{1}{3}$ part of that arch, that is to say, the subtense of the arch of 10. degrees AB, be demaunded.

The subtense given A D, is	5176381.
The $\frac{1}{3}$. part thereof is	1725460.
That somewhat augmented, is	1730000.
Or	1740000.
Or	1730000.
Let the first position be	1730000.
Whereby the subtense of the triple arch A D, sought for according to the doctrine of the third Problem, shall be	5138223,
But it should be	5176381.
Therefore it is too little by	38158.
Let the second Position bee	1740000.
Whereby the subtense sought for of the triple arch A D, according to the doctrine of the third Problem, shall be	5167320.

But

But it should have been ———— 5176381.

Therefore it is too little by ——— 9061.

Now according to the doctrine of the Rule of false position, Multiply crosse-wayes, the first Number too little: by the second position, and the second Number too little by the first position. And because the sine lesse, is to both the Numbers, subtract the Products one from another, and you shall have the number to be devided after this manner.

The first Product is ———— 66394920000.

The second Product is ———— 15675530000.

The Dividend is ———— 50719390000.

In like manner, subtract the one lesse from the other lesse number, and you shall have the Divisor, after this manner.

The one lesse ———— 38158.

The other lesse is ———— 9061. subtracted.

Resteth the Divisor ———— 29097.

The Division it selfe.

The Dividend, 50719390000

The Divisor, 29097(1—1

216223(7—8

29097

203679

125449(4—3

39097

116388 7/7

09610(3—6 1/7 0

29097.

87291

33190(1—7

29097

40930(1—8

29097 7/7

118330(4—3%0

29097 7

116388

1942

The

The diuision being ended, the quotient, as you see, is the Number 1743114. for the subtense A B. By which Number worke againe as by the first and second position, and the quotient will bee againe too little, but very little, to wit, 3. Therefore take a number a little greater then the number 1743114. to say the Number 1743115. and repeating the former worke, you shall finde the subtense A D, such as the giuen subtense was in the beginning: that is 5176381. which notwithstanding in the conclusion, is more then the truth. And therefore also, the subtense 1743115. in the conclusion will bee more then the truth. yet more neere the truth then the subtense 1743114. as by the worke appeareth; because, truely it leaveth no apparent difference between the subtense given A D, and the subtense sought for.

Note. The rule of false alwayes sheweth you the truth in the least part, in twice as many more Ciphers, as the first or second position had signifying figures (as 1. 2. 3. 4. 5. 6. 7. 8 9. but not 0) towards the last. As for example: In the aforegoing example: either position severally taken, had three signifying figures towards the last, to wit, 173. or 174. Therefore the Rule of false shall exactly produce the truth in sixe Cyphers, to say in these 1743114. whence it appeareth, If you make the first position 17431140000000. the second 17431150000000. you shall in the end have the true subtense exactly to the Radius 100000000000000. But if againe, you shall increase this subtense with 14 Cyphers, and shall take the last signifying figure in the one position lesser, and in the other greater, you shall finde very exactly the true subtense demanded to the Radius: 10000000000000000000000000000. and so for the rest, so that you had the subtense A D, first given in so many parts.

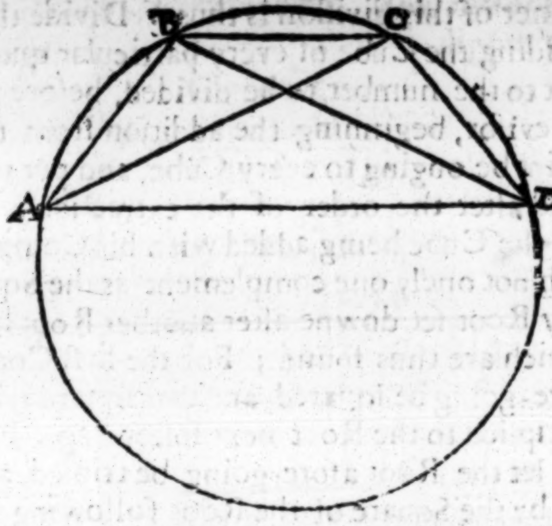
Otherwise by *Algeber.*

The Rule. *Divide the subtense given by 3. l. — 1 c. the quotient shall be the subtense of the third part of the arch given.*

The Reason of the Rule. For the subtense of what soever arch is equall to 3. Rootes lesse 1. Cube, of which Rootes, one Roote is the subtense of the third part of that arch.

Which is thus demonstrated: Let the subtense given, bee AD, beeing the subtense of the arch A B C D, and let the subtense

A B, or



A B, or B C, or C D, being the subtense of the third part of that arch, be sought for: Let B C the subtense of the third part be put for 1.l. Then the subtenses of the double arches, to wit, the right lines A C, and B D, every of them shall be to the arch B C. l. 4 q. —— 1. bq. by the demonstration of the Problem aforegoing. But the figure ABCD, is a foure-sided figure inscribed in a circle, and intersected with Diagonals. Therefore the right angled figure made of the Diagonals A C, and BD. is equall to the right angled figures (of the sides) added together by the 54. of the first: First then I multiply the Diagonals together, and thereof is made the square 4.q. ——1.bq.) For to multiply a surd Number by it selfe, is nothing else but to take away the signe l. (After I multiply the side *A B*, by the side CD. that is 1.l. by 1.l. and thereof is made 1q. which I subtract out of the square of the Diagonals, that is out of 4q.——1 bq. & the rest is 3q——1 bq. for the right angled figure made of BC, & AD, wch right angled figure 3 q.—1 bq. if I divide by the side BC. that is by 1 l. the quotient is the side *AD*, 3 l.——1 c. Therfore 3l,--1 c. one of whose sides being the subtense of 1. third part, is equall to the subtense of the arch giuen; and consequently if the subtense of the given arch be divided by 3 l ——, 1 c, the quotient shall bee the subtense of one third part which was to bee demonstrated.

The

The manner of this division is thus; Divide the Subtense given, by 3, adding the Cube of every particular quotient with his Complement to the number to be divided, before you move forwards the Devisor, beginning the addition from the right hand, under the point belonging to every Cube, and put to the Number to bee devided after the order of the extraction of the Cubicke Root: I say the Cube being added with his Complements; For the Cube hath not onely one complement as the Square, but the Cube of every Root, set downe after another Root hath two complements, which are thus found; For the first Complement, let the Root afore-going be squared, and that square tripled, and that triple be multiplied to the Root next following. For the second complement, let the Root afore-going be tripled, and that triple be multiplied by the Square of the Root following, as the worke of the Example following, which I have set downe at large, sheweth.

Example.

The Subtense of 30 degr. given, from whence the subtense of 10 deg. is to be drawne.

0	517	638	1			
(3)	(3)1	Adde (01				
218	638	(7				
(3)3	913	Ad.				
12	551	100	(4			
(3)	355	024	Ad.			
906	124	000	(3			
(3)27	295	407	Ad.			
33	419	407	000	(1		
(3)	911	466	991	Ad.		
4	330	873	991	(1		
(3)	91	152	451	231	Ad.	
1	026	442	231	(4		
(3)36	461	273	334	544	Ad.	
258	487	715	565	554.	Ad. There should follow (8)	
(3)						

The

The particular Squares of the Subtense of 10. degrees.

0· 1
01
001 · 7
27
189
00289· 4
344
1376
0030276 3
3483
10449
003038049. 1
34861
0030383976I· I
348621
00303843247 21· 4
3486124
12944896
00303844641 6996

The particular Cubes of the Subtense of 10. degrees.

C.0001. (1
3913
C.0004913 (7
355024
C.0005268024 (4
27295407
C.000529531 9407 (3
911466991
C.00052962308 73991 (1
91152451231
C.000529632202644 2231 (1
3646127333 4544
C.000529635848 7715565544 (4

l. 1
q. 1
3
3
l. 7
1.Comple. 21
2.Comple. 147
The Cube 343
3913. the
Cube of the Roote 7.
with his complements.

The finding out of the particular Cubes of the Subtense of 10 degrees.

l. 17
q. 289
3
867
l. 4
1.Compl. 3468
2.Compl. 816
The Cube, 64

l. 17
3
51
q. 16
306
51
816

l. 4
q. 16
C: 64

355024. the Cube of the
Root 4. with his Complements.

l. 174
g. 30276
3
90828
l. 3
272484
4698
27
27295407

l. 174
3
522
q 9
4698

l. 3
q 9
C. 27

l. 1743
q. 3038049
3
9114147
5229
1
9114669 91

l. 1743
5229

l. 17431
q. 303839761
3
911519283
52293
1
911524512 31

l. 17431
3
52293

l. 174311
q. 30384324721
3
91152974163
4
364611896652
8366928
64
364612733 34544

l. 174311
3
522933
q. 16
3137598
522933
8366928

l. 4
q 16
c. 64

The proofe of the afore-going worke, by composition contrarily.

1 l.	01743114	8	The Subtense of 10 degrees.
3 l.	05229344	4	
1 c.	00052963	5	Subtract.
	05176380	9	Remaineth for the subtense of 30 degr.

The

The seventh Problem.

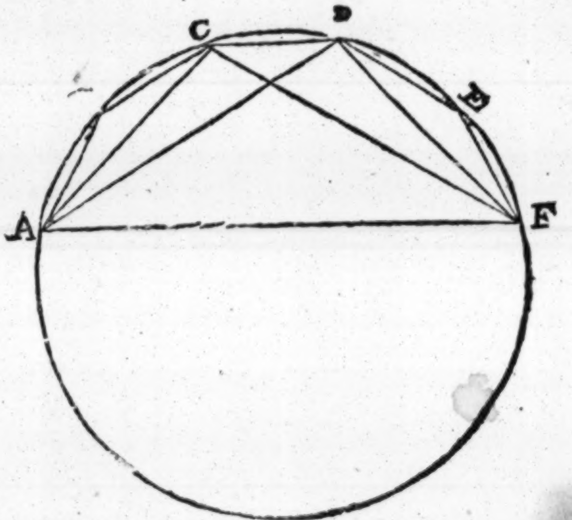
36 *The subtense of any Arch being given, to find the subtense of the fift part of that arch.*

The Rule. *Put for the subtense demaunded, somewhat more then the $\frac{1}{5}$ part of the subtense given, and thereby find out the subtense given, according to the doctrine of the fourth Problem: Whereby if you find the same, you have your desire : But if otherwise, note the difference by more or lesse, and the same worke repeat by another Position of the subtense demanded : Againe, note more or lesse ; And lastly find out the truth by the rule of false Position, as in the 6 Problem.*

The reason of the rule. Is the same which was in the sixth Problem.

Example.

Let A F, 1743115. the subtense of 10 degrees be given : And let the subtense of the fift part, that is the subtense of two degrees C D, be demanded.



The subtense of 10 deg. is ———— 1743115 almost,
The $\frac{1}{5}$ thereof is ———————— 348623

Let the first position be ———— 349000
Thereby A F, is found ———— 1742875
But it should be ———————— 1743115

 These

Therefore it is lesse by ——— 240

Let the second Position be ——— 349100
Thereby A F, shall be ——— 1743373
But it ought to be ——— 1743115

Therefore it is more — + 258
To which adde the lesse — 240

And you shall have for the Divisor ——— 498.

Then multiply crosse-wayes, that is the lesse by the greater position: And the more by the lesser position. And then
The first product shall be ——— 83714000
The second Product shall be — 90042000

These adde and you shall have the Divisor 17826000
The divisor was — 498 (3 — 3
1494

2444
498 (4 — 7
1992

The Quotient that is the true subtense of two degrees C D, is precisely 349048.

4506
498 (90 — 7
4482

02400
498 (4 — 2
1992

4080
498 (8 — 1 + 3
3984 0

96.

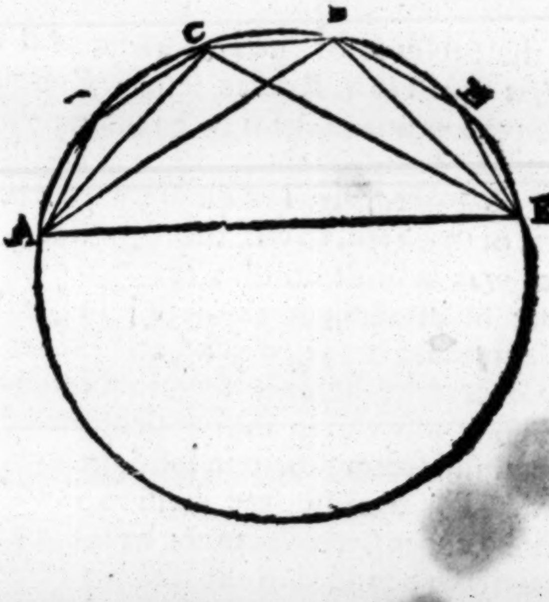
Otherwise by *Algebra.*

The Rule: *Divide the subtense given by* 5 l. — 5 c. + 1. ss. *the Quotient shall be the subtense of the fift part of the Arch given.*

The

The reason of the Rule. For the subtense of any Arch whatsoever, is equall to five rootes, lesse, five cubes more one solide; one of whose rootes is the subtense of the fifth part of that Arch. Which is thus demonstrated.

Let A F, the subtense of the arch A B F, be giuen, and let the subtense of the fifth part of the arch A B F, be demanded, that is the subtense of the arch C D; to wit, the right line CD. Let CD be put for one Root: or which is all one, Let C D, be 1l. Therefore A C, shall be l 4q, — 1bq. And likewise D F, by the demonstration of the 31. Problem. But A D, shall be 3l: — 1 C. And so also CF, by the demonstration of the Problem next aforegoing. Now in the quadrilateral figure AC D F, intersected by the Diagonals A D, and C F, the right angled figure of the diagonals A D, and C F, is equall to the right angled figures (made of the opposite sides; to wit, of CD, and AF and also of A C, and D F, (added together by the 54. of the first.



First, then I multiply the Diagonals A D and C F, together, Then I multiply the sides opposite, AC and D F, together: And I subtract the product of this multiplication, from the product of the Diagonals. The remainder is the right angled figure, of the other two opposite sides C D, and A F, which right angled

 figure

figure divided by the right line C D, leaveth the right line A F,

The whole Algebraicall worke is thus:

AD, 3 l — 1 C.
CF, 3 l — 1 C.

9 q — 3bq.
— 3bq. † 1 q C.

AD, CF, 9 q — 6bq ‡ 1 b C

AC, 1 4 q — 1bq.
DF, 1 4 q — 1bq.

AC, DF, 4q — 1bq.

AD, CF, 9 q — 6bq ‡ 1 b C
AC, D F, 4 q — 1 bq.

CD, A F, 5 q — 5 bq. ‡ 1qC
The Devisor C D, 1 l.

The Quotient A F, 5 l, — 5 C ‡ 1 ss.

Therefore the subtense of the arch given *A B F*, to wit, the right line *A F*, is equall to 5. Rootes, lesse 5. Cubes, more by one solid: of which rootes one of them is the subtense of the fift part of the arch given. And consequently, if I shall divide *AF*. the subtense given by 5l. — 5c. † 1ss. The quotient shall be the subtense of the fifth part of that arch, to wit, the right line A B or B C, or C D. &c, which was to be demonstrated.

The manner of dividing by 5l. — 5c † 1 ss, is thus: First, of all the points agreeable to the cubicke, and solid roots are to bee put ouer the number to be diuided: then the Number diuidend is divided by 5. adding alwayes to the quotient found 5 cubes with his complements, and subtracting one solid, before the devisor be moved forward. For because the division must be made by 5l - 5c. therefore 5 cubes (which cannot be taken from the devisor) are contrarily to be added to the number to be divided. In like manner, because the division must be made by 5 † 1 ss. Therefore one solid (which cannot be added to the divisor) is to be subtracted from the number to be divided.

Example

Example: *Let the subtense of* 10. *deg. be given, out of which is to bee extracted the subtense of* 2. *Degrees.*

```
S.
C.  2c.   S|C.
0. 17.4331|1           4|
5)(5)(5)13|5 Adde       |   (3
24      44|6 400       0|
          |     24     3|Subtract.
  24 44   |6.  375     7|
  (5)   6 |1   529   Ad |   (4
  4    50 |7   895     7|00   000|
          |     21     1|35   424|Subtr.
 (4)(5)0  |7   874     5|64   576|
  (5)   1 |6   022     7|45 Adde |   (9
        2 |3   897     3|09   576|000 00|
          |      6     3|30   413|767 49|Subtr.
        2 |3   890     9|69   162|232 51|
     (5)  |(5)  73     0|88   970|320 Ad|   (4
          |3   964     0|58   138|552 51|00000|00000|
          |             |29   677|769 34|94110|73024|Subtr.
          |3   964     0|28   460|783 16|05899|26976|
          |    (5)      |                        ( 8 almost
```

The particular Squares of the subtense of 2. *degrees.*

```
00 (3
00003
00009
0000 (4
    64
   256
0001156 (9
    689
   6201(0
```

```
  00012180100 (4
            69804
            219216
  ----------------------
  00121 8289216 (8
              698088
              5584704
  ----------------------
          1218345063O4
```

The particuler Subtenses of the Cube of two degrees.

```
C.0000027  Radix ( 3
       12  304
-----------------------
C.0000039  304        (4
        3  204. 549
-----------------------
C.0000042  508 449         (90
           14 617 795 264
-----------------------
C. 0000042  523 166 795 264   (4
            2 923 961 134 592
-----------------------
C. 0000042  526 090 756 398 592. one Cube (8
                              5. the Multiplyer.
-----------------------
C. 00000212. 630. 453. 781. 992. 260. 5. Cubes.
```

The particuler Solides of the subtense of two degrees.

```
00000000243 —the Root ( 3
        211    35424
-----------------------
        454    35424              (4
         63    40413. 76749
-----------------------
        517    75837. 76749,   (90
               29677. 76934. 94110. 73024
-----------------------
        518    05515. 53683. 94110. 73024 (4
               5937. 18660. 97927. 80583 &c.
-----------------------
00000000518    114521 72344. 92038. 53607.     (8
```

The

The proofe of the Resolution made by contrary Composition:

1. l. 00349048. The subtense of two degrees.

5. l. 01745240.
— 5. c. 00002126. Subtract.

01743114.
4 1. ss. 000000005 1, &c. Adde if any thing be to be added.

01743114. The totall is 46 the subtense of 10. deg.

But how the particuler Cubes are to be found, was shewed in the example, of the afore-going Problem, Nor is this any new thing, but that euery particuler Cube, be multiplyed by 5. before they be added, because here 5. Cubes are to be added to the Number to be divided: As for example, the first particuler Cube in this example was 27. this multiplyed by 5. maketh 135. The other particuler Cube with his complements, was 12304. This number multiplyed by 5. maketh 61520. And so forwards.

The particuler Solides, you shall finde thus: A solide is made by the multiplication of a Cube, by a square: As the solide of 3. is thus made: three times 3. is nine, and thrice 9. is 27. and 9. times 27. is 243. And this is the making of the solide of one figure, or else of more figures, considered joyntly together: As the solide of 34. is 45435424. For the square of 34. is 1156.: the cube is 39304, which two multiplyed together, make the Number 45435424. But if you would finde out the solides of euery figure with their Complements severally, that is to say: If after the finding of the solide of 3. you would finde the solide of 4. which, with his Complements added to the solide of 3. maketh the solide of 34. you must worke another way, as thus: The solide of more figures, as for example: the solide of 34. to all the figures after the first, hath foure complements: as there are foure figures put betweene euery of the points of the solide Rootes: those foure complements you shall thus find; For the first Complement you shall first square the square of the Root afore-going, then

then you shall multiply that biquadrat by 5. And that product you shall multiply by the present roote. For the second complement you shall cube the roote aforegoing, and multiply that number, by 10. and that last product you shall multiply by the square of the present roote. For the third complement, you shall square the roote aforegoing, and multiply that square by 10. And that product you shall multiply by the cube, of the present roote. For the fourth complement, you shall multiply the roote aforegoing, by 5. And that product you shall multiply by the biquadrat of the present roote; Then multiply the present roote into a sursolide; And lastly adde those 5 Numbers together, under writing one under another in such manner as the example following sheweth.

The Solide of 4. in the Roote 34.

	Pc. l. 3.	pc. l. 3.	pc. l. 3.	pc. l. 3.	l 4
	q 9.	q 9.	q 9.	5.	16.
	bq 81.	c. 27.			
	5.				
	Dec. 270.	Dec. 90.	15.		c. 64
	405. q. 16.	c 64. bq. 256.			64
1 *Complement*, 1620.	l 4. 1620.	360.	90.		90
2 *Complement*, 4320.	207:	140.	75.		1024.
3 *Complement*, 5760.	4320.	5760	30. is		
4 *Complement*, 3840.			3840.		
ss. of 4. is.	1024.				

21135424. The particuler solide of 4. with his Complements in respect of 3. the Roote afore-going.

Note. By the same manner of worke, you may find the subtense of the seventh, ninth, eleventh, thirteenth, and infinitly of any part whatsoever of any uneven Number, if need require.

The eight Problem.

The sines of two unequall arches, being given together with the sines of their complements, to find the sine of the summe, or of the difference of those arches.

The

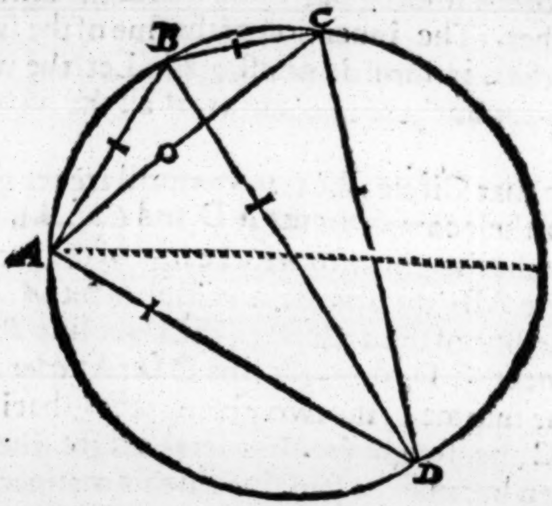
The rule. *Multiply alternatly the sine of the one arch by the sine of the complement of the other: If you adde the products together and divide that total by the* Radius, *by cutting of 7 figures from the right hand, you shall have the sine of the summe of the two given arches: But if you subtract the lesser product from the greater, you shall have the sine of the difference of these arches.*

The reason of the Rule. This rule hath two Members, one is of the finding out of the sines of the summe of two vnequall arches: The other is of the finding out of the sine of the difference of two vnequall arches. The invention of the sine of the summe of two vnequall arches, is thus demonstrated. Let the whole Circle, *A B C D*, be put onely of 180, parts that all the subtenses may be as the sines.

And let in that Circle, the two vnequall arches given, be *A B* and *B C*, and their complements A D and *C D*. And let the sines of all these arches be given, to wit, the sine of the arch AB, let be the right line AB: the sine of the complement *A D*, the right line *AD*: the sine of the arch, *B C*, the right line *B C*: the sine of the complement C D, the right line C D, And let A C, which is the sine of the summe of the two given arches, that is of the arches, A B and B C, be sought for. But let the right line B D, bee the *Radius*. Then because the said sines in this manner inscribed in a Circle make a quadrilaterall figure, intersected with diagonals, in which figure the right angled figure made of the diagonals, is equall to the two right angled figures, added together made of the two opposite sides by the 54. of the first. Therefore if you multiply the sine A B, by the sine of the complement of the arch B C; that is, by the opposite side C D; And likewise the sine of the arch B C; by the sine of the complement of the arch, AB; that is by the opposite side A D; and adde those two products together, you shall have a right angled figure, equall to the right angled figure of the diagonals AC, and B D. which right angled figure if you divide by the knowne side, to wit, by the *Radius* B D the quotient shall produce the vnknowne side AC, to be the sine of the summe of the arch *A B* and BC, or the sine of the arch AC: which was to be demonstrated.

The invention of the sine of the difference of two vnequall arches,

arches, is thus demonstrated. Let the sine of the greater arch *A B* be the right line *A B.* and the sine of the lesser arch *B C*, be the right line *B C*. Let the sine of the difference *A C*, bee the right line *A C*: the sine of the Complement of the greater arch *A B*, let be the right line *A D*. And the sine of the complement of the lesser arch *B C*, be the right line *C B*: B D being the *Radius*. Then againe, because the figure *A B C D*, is a quadrilaterall figure inscribed in the Circle, and intersected with diagonals.

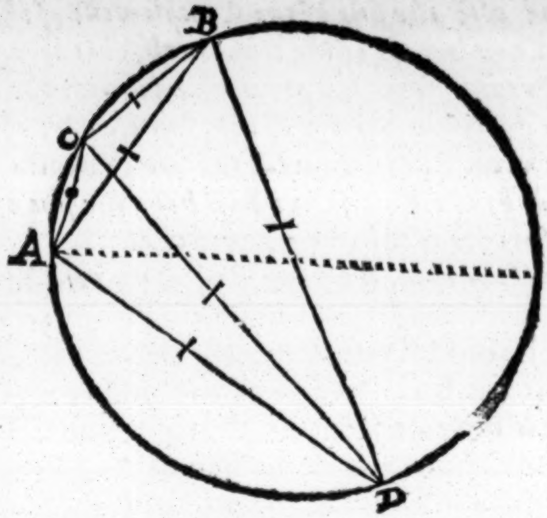


Therefore if I multiply the sine of the greater arch *A B*, that is the right line *A B*, by the sine of the complement of the lesser arch *B C*, that is by the right line *C D*: I shall haue a right angled figure made of the diagonals, equall to the two right angled figures made of the two opposite sides.

Moreouer, if J multiply the sine of the lesser arch *B C*, to wit, the right line *B C*, by the sine of the Complement of the greater arch *A B*; to wit by the right line *A D*: And if J take this product of the two opposite sides *B C*, and *A D*, from the right angled figure made of the diagonall *A B* and *C D*.

There shall remaine the right angled figure, made of the two opposite sides *A C*, and B D: which right angled figure, if I diuide by the knowne side B D, the *Quotient* shall produce the side vnknowne A C, being the sine of the Difference of the two vnequall arches giuen A B, and B C, which was also to bee demonstrated.

Example



Example of both Members.

Let the greater arch *A B*, be ———	20. deg.
The lesser arch *B C*, be ———	15. deg.
The summe of these two arches shal be	35. deg.
Their difference ———	05. deg.

The sines of the arches given, and of their Complements are.

The sine of the arch *A B*, is ———	3420241
The sine of the Complement *A D*, —	9396926
The sine of the arch *B C*, is ———	2588190
The sine of the Complement *C D*, —	9659258

Then let them be multiplyed alternatly, the sine of the arch A B, by the sine of the Complement of the arch B C; to wit, by C D. And the sine of the arch B C, by the sine of the complement of the arch A B, that is by A D: And

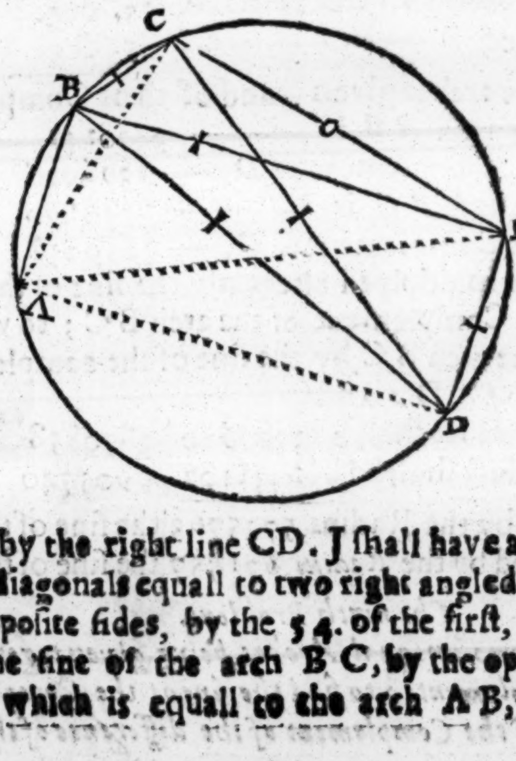
The greater product shall be	3303660	3870858
The lesser product shall be -	2432102	9905740
The sum divided by the Radius,	5735763	the sine of the sum 35d.
The differ. divided by the *Radius*,	871557	the sine of the diff. 5.d.

The ninth Problem.

38 *The sines of two unequal Arches, being given together with the sines of their Complements: to find the sine of the Complement of the totall, or the sine of the Complement of the difference of those Arches.*

The Rule. *Multiply the sine of the one Arch by the sine of the other arch. And also the sine of the Complement of the one arch, by the sine of the Complement of the other arch, this being done: if you take the lesser Product from the greater, and divide the Remainder by the Radius, you shall have the sine of the Complement of the summe of the two given arches. But if you adde the two Products together, and divide that total by the Radius, you shall have the sine of the Complement of the difference of the two given arches.*

The reason of the rule. And this Rule hath two Members. The first is thus demonstrated. Let the sine of the greater arch, be the right line A B, or D E equall thereunto; the sine of the complement, the right line B E; the sine of the lesser arch, the right line B C, the sine of the complement, the right line C D, the sine of the summe, the right line A C. the sine of the complement of the summe, the right line C E. the *Radius*, the right line B D. Then because the figure B C E D, is a quadrilaterall figure inscribed in a Circle, and intersected with diagonals: Therefore if I multiply the sine of the complement of the arch A B. to wit, the right line B E, by the sine of the complement of the arch

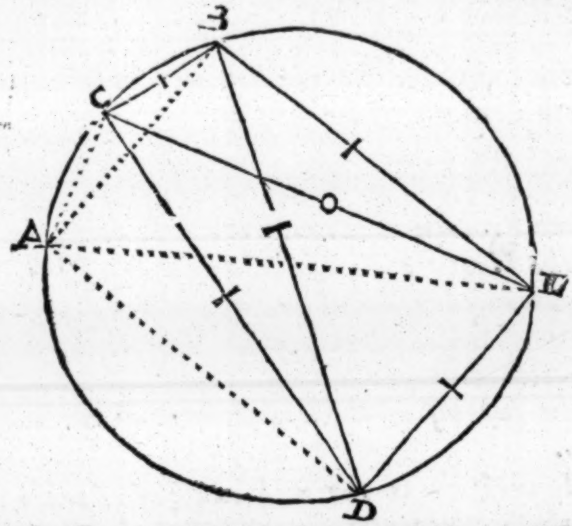


BC, to wit, by the right line CD. I shall have a right angled figure of the diagonals equall to two right angled figures, made of the two opposite sides, by the 54. of the first, Then againe if I multiply the sine of the arch B C, by the opposite sine of the arch D E, which is equall to the arch A B, and subtract this

right

right angled figure, from the right angled figure of the diagonals, the Remainder shall be the right angled figure, made of the two other opposite sides BD, and C E. which right angled figure if I diuide by the knowne side, to wit, by the *Radius* B D. the quotient shall produce the vnknowne side C E, the sine of the complement of the summe of the 2. giuen arches A B, and B C, which was to be demonstrated.

The latter member is thus demonstrated. Againe let the sine of the greater arch, be the right line A B, or E D: the sine of the complement, the right line BE, the sine of the lesser arch B C, the sine of the complement C D, the sine of the difference A C, the sine of the complement of the difference C E: the *Radius* B D.



Then because the figure CBED. is a quadrilaterall figure, inscribed in a circle, and intersected with diagonall lines, Therefore if I multiply euery of the two opposite sides, to wit, the sine B C, by the sine D E, and the sine of the complement B E, by the sine of the complement C D, and adde their products together, I shall haue the right angled figure, made of the diagonall CE, and BD, which if I divide by the knowne side, to wit, by the *Radius* B D, the quotient shall produce the vnknowne side C E, the sine of the complement of the difference of the 2. giuen arches AB, and B C: to wit, the sine of the complement of the arch A C, which also was to be demonstrated.

Example

Example of both Members.

Againe, let the greater Arch A B, be —— 20. deg.
The lesser arch *B C*, be —— 15 deg.
The summe or totall, is —— 35 deg.
The difference, is —— 5 deg.

The sines of the giuen arches, and their Complements as before.
The sine, *A B*, 3420201. The sine *A D*, or *B E*, 9396926
The sine *B C*, 2588190. The sine *C D*, —— 9659258

Let, *A B*, be multiplied by *B C*, and *B E* by *C D*, and the products subtracted from, and added one to another. And diuided by the *Radius*.

The greater Product shall be ——	9076733	2640908
The lesser Product shall be ——	885213	0026190

The difference diuided by the *Radius*, is 8191520. being the sine of the Complement, of the summe.

And the summe diuided by the *Radius*, is 9961946. being the sine of the Complement, of the difference.

39 *These nine Problems are as it were Instruments, by whose helpe all the rest of the Sines are drawne out of the totall sine.*

The most commodious order of finding of them is thus; First, are to be found out the subtenses of the Arch, of 60. deg. 30 d. 10, d, 2, d. 1. d. 20. min. 10. m. 2. m. 1. m. 20. sec. 10, s. 2. s. *by the* 5 6, & 7. *Problems.*

Likewise the subtens of the Complement of those Arches, by the first Problem.

For this Inquisition is the most exact of all other: that you may rightly call those Subtenses, principles of the Canon of Triangles. Then out of the halfe of those subtenses, that is, out of the Sines of the arches, of 30. d. 15. d. 5. d. 1. deg. 30. m, 10 m. 5. 1. min 30. s. 10. s. 5. s. 1. sec. *together with the sines of the Complements of those Arches you shall easily find out all the Sines, by the second,* 8. *and* 9. *Problems. By the second Problem, by finding out the sine of* 2. deg *also of* 2. min. *and of* 2 sec. *or* 26. sec. *By the* 8. *and* 9. *by continually adding to the sines hitherto found, the sine of* 1. deg. *or of* 1. min. *or of* 10. sec. *or also of* 1. *second: as you would haue the Table briefe, or more ample.*

I found out the subtenses aforesaid, of 60. 30. 15, &c. after the same manner, as I haue set downe in the parts of the *Radius*, of 1000000000000000000000000000. And so I found them as followeth.

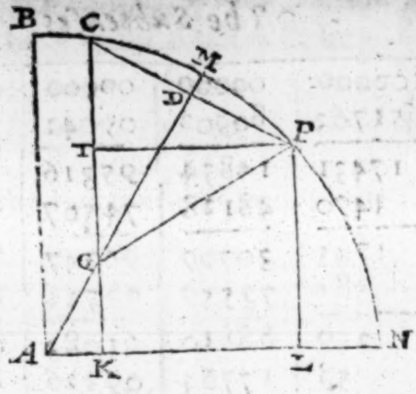
The

The Arches.			*The Subtenses.*				
60 d	00. m.	00 .s.	100000	00000	00000	00000	00000
30.	00	00	51763	80902	05041	52469	77977
10.	00	00	17431	14854	95316	34711	61285
2.	00	00	3490	48128	74567	02563	88379
1.	00	00	1745	30709	96747	86992	97569
	20	00	581	77559	68723	86874	86923
	10	00	290	88810	61083	07015	25490
	2	00	58	17764	09126	84919	27486
	1	00	29	08882	07640	17437	29548
		20	9	69627	36183	92296	70823
		10	4	84813	68106	20557	04030
		2	0	96962	73622	15273	56399
The halfe of the Arches.			*The halfe of the Subtenses.*				
30 d.	00. m.	00. s.	50000	00000	00000	00000	00000
15	00	00	25881	90451	02520	76234	88988
5.	00	00	8715	57427	47658	17355	80642
1.	00	00	1745	24064	37283	51281	94189
	30	—	872	65354	98373	93496	48884
	10	—	290	88779	84361	93442	43461
	5	—	145	44405	30541	53507	62745
	1	—	029	08882	04563	42479	63743
	00	30	014	54441	03820	07367	14774
		10	4	84813	68091	96148	35411
		5	2	42406	83053	10278	52015
		1	0	48481	36811	07636	78199

40 In this *Theorem* is an excellent Compendium of the sines. *The difference of the sines of two arches, equally distant on both sides, from 60. degrees, is equall to the sine of the distance.*

The declaration. Let C N, and P N, be the two arches equally distant from 60 d. M N, that is equally distant on both sides from the point M. And let the right lines C K, and P L, be the sines of

 those



those arches, being perpendiculers upon the right line A N, by the 3. consect of the 7. hereof. And thereupon paralell one to another by the 38. of the first.

Moreover, let the right line P T, be drawn perpendiculer upon the right line C K, paralell to the right line K L, by the 38. of the first. This right line T P, cutteth from the right line C K, another line T K, equall unto P L, by the 39 of the first. And leaveth the right line T C, for the difference of the sines C K, and P L.

Lastly, the sines of the distance, of either of them from 60. deg. Let be the right line C D, or D P. I say, that the right line TC, is equall to the right line C D, or D P.

The Demonstration. For because in the Triangle G C P, that the perpendiculer G D, doth bisect the base C P, by the 12. hereof, and by the Pro: Therefore the sides G C, and G P, are equall by the 23. of the first. And the angles C G D, and D G P, are also equall by the same; and lastly, the angles G C P and G P C, are likewise equall, by the 26. of the first. But the angle C G D, is 30. deg. for that it is equall to the angle B A M, by the 38. of the first.

Therefore the angle C G P, is 60. deg. for that it is double to the angle C G D.

But because the angle CGP, is 60. deg. therefore the other two angles G C P, and G P C, added together, are 120 deg. by the 49. of the first.

But these other two are demonstrated to be equall, therefore every of them is 60. deg.

And the angle C P G, is also so many degrees, therefore the triangle

triangle C G P, is equiangled. But because the triangle C G P is equiangled, therefore also it is equilaterall by the 28. of the first.

Moreouer, because the triangle C G P, is equilaterall, therefore the perpendiculer P T, bysecteth the base CG, by the 23, of the first.

Then the sides C P, and C G, are equall.

Therefore also their bisegments C T, and C D, are equall, which was to be demonstrated.

Consectarie. The sines of whatsoeuer 60 degrees, being giuen, you may find the sines of the other, 30. degrees by addition or subtraction onely.

The Illustration by Numbers. Let the arches C N, be 70: deg. PN, 50, deg. C M, or P M, 10. deg. for so many degrees are the arches of 70. deg. and 50 deg. distant from the arch of 60. deg on both sides. And let first the sines of 70, deg, and 10, d. be giuen; And let the sine of 50. degrees be demanded.

From the sine of 70. degrees C K,	9396926
Subtract the sine of 10. deg. C D, or C T,	1736482
The remainder will be the sine of 50: deg. T K, or P L,	7660444

Then let the sine of 70, deg. and 50 d. be giuen, And let the sine of 10. degrees be demanded.

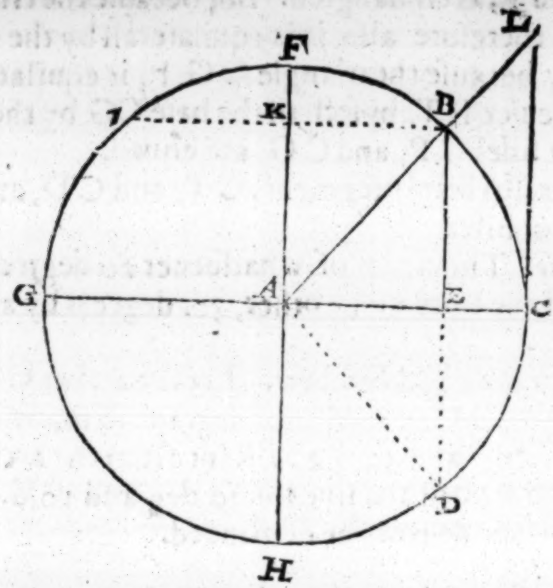
From the sine of 70. deg. C K,	9396926
Subtract the sine of 50, deg, T K, or P L,	7660444
The remainder will be the sine of 10, d, T C, or C D,	1736482

Lastly, let the sines of 50. deg, and 10, deg. be giuen. and let the sine of 70, deg. be demanded.

To the sine of 50. deg. P L, or T K,	7660444
Adde the sine of 10, deg. D P, or T C,	1736482
The whole will be the sine of 70, deg. C K,	9396926

41. *And thus farre of the making of the tables of right sines, The tables of versed sines are not needfull, as aforesaid.*

42 *The tables of Tangents and Secants, are thus made out of the tables of right sines:*



1 As the sine of the complement to the sine of an arch : So is the *Radius* to the Tangent, of that arch.

2 As the sine of the complement to the *Radius* : So is the *Radius* to the *Secant*, of that arch.

For by the 46. of the first.

1 As A E, to E B, So is A C, to C L.

2 As A E, to A B, So is A C, to A L. As for example. Let the *Tangent* and *Secant*, of the arch B C, 32. deg. be sought for : The sine of 30. deg. is 5000000. B E.

The sine of the complement, 60. deg. is 8660254. A E, Then I say.

1 As A E, 8660254. to B E. 5000000: So is A C, 10000000 to C L, 5773503. Therefore the *Tangent*, of the arch, of 30. deg. is 5773503.

2 As A E. 8660254. is to A B, 10000000. So is A C. 10000000. to A L, 11547005. Therefore the *Secant* of the arch of 30. deg. is 11547005.

43 *The briefe Rules of the* Tangents *and* Secants, *are excellent in these three Theorems following.*

The first Theorem. The difference of the Tangents, *of any two arches, making a Quadrant, are double to the* Tangent, *of the difference of those arches.*

The

whether the question be of an arch more or lesse then a Semi-quadrant, you may presently over against it find the complement thereof. And the Sine, Tangents and Secants, of the arches lesse then a Semi-quadrant, together with their arches downwards. But the Sines, Tangents, and Secants, of the arches greater then a Semi-quadrant, together with their arches doe increase ascending upwards by every minute, except in the first degree and in the Complement thereof, where I have also used one, two, or ten seconds, because otherwise the Calculation there in seconds, could not have beene without error. In stead of the differences, I have put the proportionall part either of Minutes or of tenths of seconds, for the more ease in making the Tables, I have also added the increase, wherein the unequall proportionall parts, doe increase either by every one, or by every tenne seconds, for the greater precisenesse.

I have taken divers Radusses for necessity, to wit, of 5.7, 8. 9. 10. 11. *or* 12. *figures; Which variety, the skilfull Arithmetician will easily reconcile, by using the Radius for the worke of such magnitude as every Number set downe in the table, may answer thereunto. Which that it may presently appeare, I have every where distinguished with a point put betwixt the Sines, Tangents and Secants, made for the Radius* 100000 *from the rest greater then that Radius; Nay where the Radius is more then* 10. *figures, I have put two points betwixt, whereby the Sines, Tangents and Secants of the Radius of* 10 *figures, may by a mark be discovered, and knowne from the greater Sines, Tangents and Secants: Where no Number is after the point, there the Radius is onely of five Ciphers* 100000. *as in all Tangents and Secants, of the last five degrees.*

51 *The use of this Table generally is thus: That you may readily find out the Sine, Tangent, and Secant of any arch or angle given, not exceeding* 90 *degr. together with the Sine, Tangent, and Secant of the Complement: or contrarily by the same Tables, the arch of any sine, tangent, or secant given, And so in the working of triangles, you may proceed without delay.* As if you would have the Sine, Tangent and Secant, of the arch or angle of 30. deg. or of the complement thereof: All these will be given you, in the tables according to the Radius, 10000000.

Of

Of the arch of 30 deg.	*Of the Complement:*
The Sine is—5000000.	The Sine is — 8660254.
The Tangent is, 5773503.	The Tangent is, 17320508.
The Secant is —11547005	The Secant is 20000000.

Contrarily: If 5773503. bee a Tangent giuen, and it bee demaunded what arch or angle answereth thereunto. The table will shew, that the arch or angle answering to that tangent, is 30. deg. And likewise in the other side of the table it will shew you, that the arch or angle of 60. deg. is the complement thereof.

52 *But if peradventure Seconds, be adioyned to the Minutes, and that you must vse them in the worke, then proceed as the examples following shall teach you.*

The first example. If the sine of 12. deg. 6 min. 23. sec, be to bee found, Take in the beginning of the tables the sine of 12. deg. 6. min. which is 2096186. Then gather by the proportionall part how much the remainder 23. sec will require: in saying,

10. sec, giues 474. parts, what shall 23. sec.

$$\begin{array}{r} 23 \\ \hline 1422 \\ 948 \\ \hline 1090|2 \\ \hline \end{array}$$

the sa: is 1090. parts.

Lastly, to the former giuen sine. —— 2096186.
Adde the proportionall part now found. — — 1090.

And you shall haue the sine required. —— 2097276.

The second example, If the tangent of the arch of 88, deg. 51. min. 34. sec. be demaunded: First take out of the tables the tangent of 88. deg. 51. sec. which tangent according to the Radius 100000 is 4981573. Then you shall find the proportionall part, for 34 sec, after this manner.

10. sec.

10 sec. ——— 12065 A	This adde continually first to *A* then to *B*; thirdly to *C*, & you shal have *A B C*, for 30. sec. then if you multiply *D* by 4, and cut off the last cipher, you shall have 4897 for the other 4 sec. Now adde *A B C E* together, and you shall find F.	
the increase - 00059		
10. sec. ——— 12124 B		
10. sec. ——— 12183 C		
10. sec. ——— 12241 D		
4. sec. ——— 4897 E		
34. sec. ——— 41269 F		

Adde the Tangent of 88. deg. 51. min. 4981573.

The totall, is the Tangent required, - 5022842,

The third Example. If the Tangent of 89 deg. 39. m. 24 sec. were to be found. You must thus proceed:

The tangent of 89 d. 39. m. 20. s. is	16634058, A	This adde continually to *B C*, and *D*, that you may find *C D*, & *E*. Lastly, collect *A B C D E*, into one summe.
1. sec. is ———	13425 B	
The increase is ———	00022	
1. sec. is ———	13447 C	
1. sec. is ———	13469 D	
1. sec. is ———	13491 E	

And you shall have the Tangent required, ——— } 16687890.

Or more briefly, multiply the proportionall part of 1, second by 4. and the increase by so many vnities as are in the progression of 4. places, that is by 6. (for such is the progression of 4 places 0. 1. 2. 3. which progression are 6 unites) and you shall have the same Tangent after this manner.

The Tangent is ——— 96634058 A.
1. sec. is — 13425, which multiply
by 4. is — 53700 B.
The increase ——— 00022 which multiply
by 6. maketh — 00132 C. *Adde A B C*, together, and you shall have — 16687890. for the Tangent desired, according to the *Radius*, 100000.

53 *But if contrarely any sine, tangent or secant were given, whose arch you would also find in seconds precisely. So proceed as the examples following will teach you.*

The first example. If 2097276 were given for a sine, the Radius being 10000000. And it were demanded what arch were answerable thereunto?

First, seeke out in the tables, the next lesser sine, and subtract that from the sine given, and note the arch agreeable thereunto: Then out of the Remainder you shall collect the seconds after this manner.

The sine given is ——— 2097276.
The lesser sine next unto it, is 2096186. of the arch 12 d. 6. sec.

Which subtracted, the Remainder is — 1090
10. sec. in the table is answerable to — 474
Now if 474. give 10. sec. what shall — 1090 give?
10900
474 (2

Therefore the arch sought for, is — 948 answer 23, almost
12. deg. 6. min. 23. sec. almost 1420.
474 (3 almost
1422

The second example.

If 5022839. according to the Radius 100000. be given. And the arch answering thereunto were demaunded: First againe find in the tables the next lesser tangent, and the arch answering thereunto. Then subtract that lesser tangent, from the tangent given, and out of the Remainder you shall gather the seconds after this manner.

The tangent given, is 5022839.
The lesser tangent next 4981573. of the arch of 88. d. 51. m.
The Remainder ——— 41266.
Subtract ——— 12065 the parts for 10. sec.

The Remainder is — 29201. From whence

10. sec.

10 sec. gives 12069 — 29201.
The increase 00059 — 12124. 10 seconds.
10 sec. — 12124 — 17077. remaineth: from whence
10 sec. — 12183 — 12183. the parts of 10 sec. subtracted.
10 sec. — 12242 — 4894. remaineth
1. sec. — 1224 — 1224, the parts for 01. seconds.

Now 1224 — 4896. is in 4894. almost 4. times: for foure times 1224. maketh 4896: Therefore the arch answerable to the Tangent given, is 88 deg. 51 min. 34 sec.

The third Example.

Let the Tangent 16687890. be given, according to the *Radius* 100000. And let it bee demanded what arch is answerable thereunto: You shall proceed in this order.

The tangent giuen, is 16687890
The next lesser tangent 16634058. of the arch 89. d. 39. m. 20. s.
Which subtracted resteth 53832

The parts of 1. sec is — 13425 A.
The increase is — 00022, this adde to A, B, and C.
13447. B.
13469 C.
13491 D. Now adde A, B, C, & D.

The total amounts to — 53832 for 4 sec. Therefore the arch answering to the Tangent giuen, is 89 deg. 39. min. 24. seconds.

54 *By this Table, after this manner, you shall be able without any errour in the doctrine of Triangles, to worke to seconds. And in the first and last degree especially, more certainly then by* Rhæticus *his great Tables: But in all other degrees,* Rhæticus *his Tables are better; For by that you shall worke more speedily, and not onely to seconds, but also thereby you may gather the thirds and fourths exactly. Therefore if you be wise and of abilitie, be not without that Table.*

The end of the second Booke.

THE THIRD BOOKE OF TRIGONOMETRIA.

By B. P.

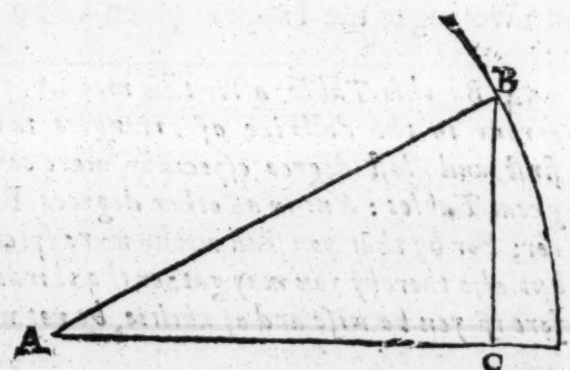
Of the dimension of Plaine Triangles.

Hitherto, I haue treated of the principles of Trigonometria, and of the necessary tables of Sines, Tangents and Secants, for the exercise thereof. Now followeth that Trigonometrie it selfe, or the measuring of Triangles, as well plaine as Sphericall. In the explaining of both which, because they are resolued onely by the Rule of proportions, as is aforesaid; First, I will set downe certaine Axiomes, *whereby may bee understood what proportions are in Triangles or parts of Triangles: Which* Axiomes, *therefore I will call the* Axiomes *of proportions, Then I will shew how those* Axiomes *are to bee vsed, or how by helpe of a few of those* Axiomes, *euery demand in any Triangle propounded by whatsoeuer three termes giuen, may quickly be found out.*

The Axiomes *of proportions in plaine Triangles are chiefly foure, being sufficient enough for euery resolution of any of them, besides the golden foundation of all Trigonometrie, which I haue explained in the first booke the 46. Proposition.*

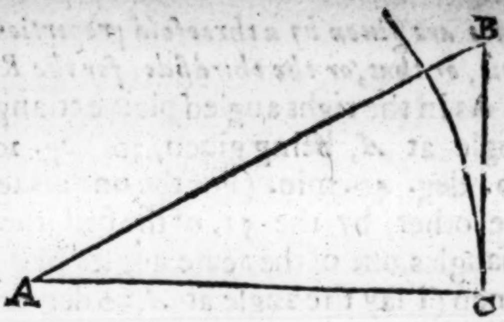
The first Axiome.

In all plaine right angled Triangles, euery side may be put for the Radius, agreeable to the doctrine of Triangles: For if you put the side subtending the right angle for Radius, the sides including the right angle, are signes of the acute angles opposite vnto them.

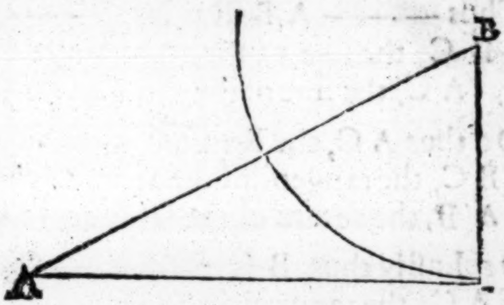


If

If you put for Radius *the greater of the sides including the right Angle, the lesser of the sides including the right angle is the Tangent, and the subtendent Line to the right angle is the Secant of the lesser acute Angle.*



If you put the lesser of the the two sides including the right angle for Radius *: the greater side including the right angle is the Tangent, and the subtendent of the right angle is the Secant of the greater acute angle.*



As in the plaine Triangle A B C. If you put the side A B, subtending the right angle for Radius, the lesser side B C, including the right angle, is the sine of the lesser opposite angle B A C, and the greater side including the right angle, is the sine of the greater opposite acute angle ABC.

But if you put A C, the greater side including the right angle for the Radius, the lesser side B C, including the right angle, is the tangent of B A C, the lesser acute angle opposite, and the subtendent A B, is the Secant of the same acute angle.

Lastly, if you put BC the lesser of the sides, including the right angle for Radius, the greater side AC, including the right angle is the tangent of A B C, the greater acute angle opposite; and the subtendent A B, is the secant of the same acute angle. All being performed by the definitions of Sines, Tangents and Secants, set downe in the Second booke.

The first Consectarie

Therefore in right angled plaine Triangles, the angles being given, the reason of the sides are also given, three wayes. And consequently. One side being given besides the three angles, every of the other

sides are giuen by a threefold proportion, that is whether you shall put this, or that, or the third side, for the Radius.

As in the right angled plaine triangle, propounded A B C. The angle at *A*, being giuen, 30. deg. 20. min. and so the angle at *B*. 59. deg. 40. min. (For the one acute angle is the complement of the other, by the 52. of the first, therefore in plaine right angled triangles, one of the acute angles being giuen, all the angles are giuen (I say the angle at *A*, 30 deg. 20. min. and at B, 59. deg. 40. min. being giuen; The reason of their sides are giuen, either

Thus —— A B, Radius —— —— 10000000
B C, the sine of the acute angle B A C, 5050298
A C, the sine of the acute angle A B C. 8631019

Or thus A C, the Radius —— —— 10000000
B C, the tangent of the acute angle —— 5851335
A B, the secant of the same acute angle 11586118

Or Lastly thus, B C, Radius —— —— 10000000
A C, the tangent of the acute angle ABC, 17090116
A B, secant of the same angle —— 19800810

That is:

It is manifested by the table of Triangles, what proportion, (for examples sake) the side A B, hath to B C, *viz.*

Either as A B, 10000000 to B C, —— 05050298
Or as -- A B, 11586118 to B C, —— 5851335
Or lastly, as A B, 19800810, to B B, 10000000. And so of the rest.

Therefore besides the angles being given after this manner, let the side A B, be given 24 feet; If it be demanded how many foot the side B C, is? I will say.

Either as A B Radius 10000000. to B C, the sine 5050298.
So is A B, the side 24 foot, to B C 12 $\frac{1207152}{10000000}$. foot.
Or as A B, the secant 11586118, is to B C the tangent 5851335.
So is the side A B, 24 foot, to B C, 12. $\frac{1398624}{11586118}$. foot.

Or lastly,

As A B, the secant of the Compl. 19800810. to B C, *Ra.* 10000000.
So is A B, the side 24 foot, to B C, 12. $\frac{2390180}{19800810}$. foot.

So if the same side A B, be given 24 foot, and that it be demanded how many foot the side A C, is: I will say,

Either as A B, *Radius* 10000000, to A C, the sine 8631019.

So

So is the side *A B.* 24. foot to *A C*, 20.$\frac{7533458}{10000000}$. foot.

Or as *A B*, the secant 11586118. to *A C*, the Radius 10000000

So is *A B*, the side 24. foot to *A C*, 20.$\frac{8577648}{11586118}$. foot.

Or lastly, as *A B*, the secant of the complement 19800810. to *A C*, the tangent 17090116.

So is A B, 24. foot to *AC*, 20.$\frac{55176584}{198008810}$. foot.

Likewise, if (the side A C, being given 20.$\frac{7144756}{10000000}$. *foot) it bee demanded how many feete the side B C, is? I will say:*

Either as A C, the sine 8631019. is to B C, (the sine 5050298

So is the side A C, 20.$\frac{7148856}{10000000}$. foot to B C, 12.$\frac{[illegible]}{8631019}$. foot;

Or as A C, the Radius 10000000. to B C, the tangent 5851335

So is A C, 20.$\frac{7144456}{10000000}$. foot, to B C. 12.$\frac{[illegible]}{[illegible]}$. foot.

Or lastly, as A C, the tangent of the Complement. 17090116 to B C, Radius, 10000000.

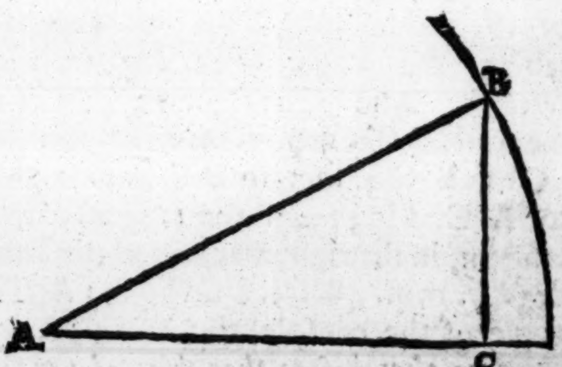
So is A C, 20.$\frac{7144456}{10000000}$. foot to B C, 12.$\frac{[illegible]}{17090116}$. foot.

Now the skilfull Arithmetician in the serious use of Trigonometria in his calculation, will alwayes frame his proportion so, that he may have the Radius in the first place, to avoyd the troublesome paines of division.

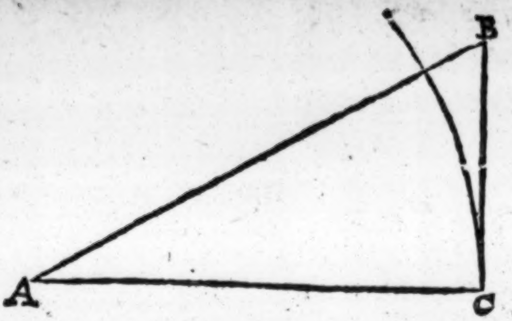
The second Consectary.

Two sides whatsoever being given to both the acute angles is given a double proportion, that is as you put the one or the other of the given sides for the Radius.

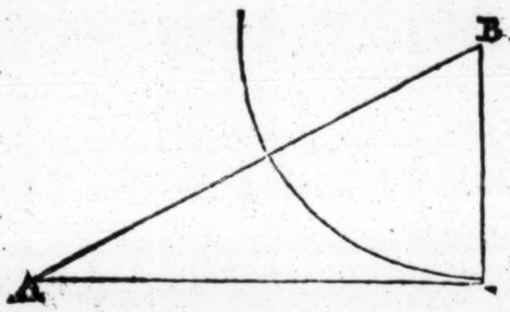
As in the plaine right angled triangle A B C, if the two sides *A B*, and B C, not including the right angle be given the one 5. and the other 3. foot. And the two acute angles A, and B, are demanded, I will say



Either as A B, 5. foote, to *B C*. 3. foote, So is *A B*, Radius 10000000. to the sine of the angle *B A C*, 6000000. to which sine in the left margent of the table answereth the angle B A C, 36. d. 52.m.12.sec.and in the right margent is the sine of the compl. *ABC*, being 53. deg.7. min.48.sec.



Or as B C, 3. foote, to *A B*, 5. foote, So is *B C*, Rad: to *A B*; 16666666 the secant of the angle A B C, to which secant in the right margent of the tables, the angle of 53, d.7. m.48.sec.answereth. And in the left margent, is the angle of the complement, being 36. deg. 52 min. 12. sec.



Likewise in the same right angled plaine triangle A B C, If the two sides A C, and B C, comprehending the right angle be given the one 4. and the other 3. foot: And the acute angles A, and B, bee demanded. I shall say

Either as *A C*, 4. foote to B C, 3. foote.

So is *A C*, Radius 10000000, to BC, 7500000. the tangent of the angle BAC, To which tangent in the left margent of the tables, the angle of 36. deg. 53. min. 12. sec. for B A C, answereth. And in the right margent is the angle of the complement being 53. deg. 7. min. 48. sec.

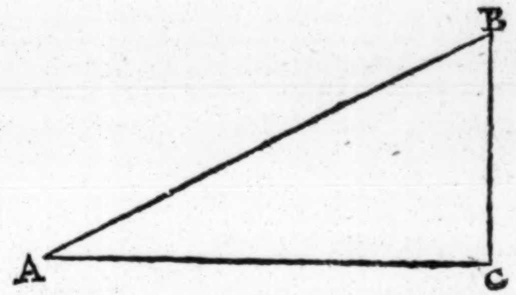
Or as B C, 3. foot, to *A C*, 4. foot. So is B C, Radius 10000000 to A C, 13333333. the tangent of the angle A B C. To which tangent, in the right margent of the Tables, the angle A B C, 53. deg. 7. min. 48. sec. answereth: And in the left margent is the angle of the complement being 36. deg. 52. min. 12. sec.

Note that before the tables of tangents were found out the two sides including the right angle being given; the acute angles and the third

third side were thus found out. First the sides A C, *and* B C, *including the right angle, were squared, and out of the somme of those 2. squares, the square root was extracted: which roote was the side* A B, *by the last Pro: but one of the first of* Euclide, *that is, by the* 50. *of my first booke.*

Then having the sine AB, *you were to say thus:*

As the side A B. *to the side* B C. *So is* A B: *Radius to* B C. *the sine of the angle* BAC, *which being knowne, the angle* A B C. *was knowne. Now wee have no need of these circumstances.*

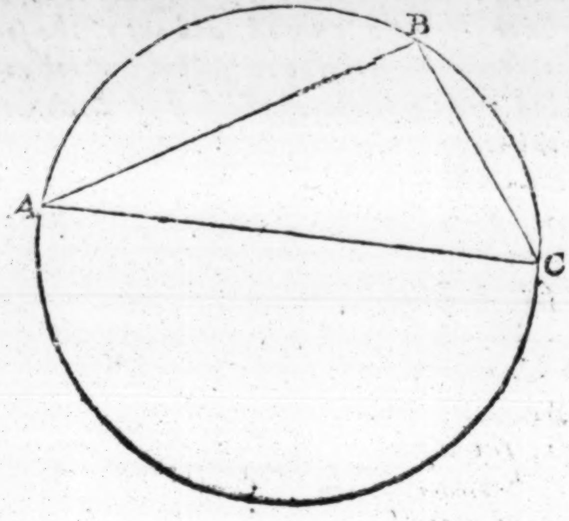


The Second Axiome.

In all plaine Triangles, the sides are in proportion one to another as the sines of the angles opposite to these sides,

For the sines are the one halfe of their subtenses: *But the sides of every plaine Triangle are in proportion one to another as the subtenses of the angles opposite to those sides.* Therefore also are the halfe of the subtenses in the same proportion. For the same reason, that is, of the whole to the whole, the same reason is of the halfe to the halfe; As in the nineteenth Pro: of the second booke I have manifested: And nature it selfe sheweth.

But that the sides of every plaine triangle are in proportion one to another, as the subtenses of the angles opposite thereunto hence appeareth; Because a circle may be circumscribed above every plaine Triangle, the Center being found out to the three points of the three angles: which being done, the sides then themselves of that plaine Triangle, are the subtenses of the angles opposite to those sides, that is, of the arches opposite to those angles, and are the double measures of them, by the 53, of the first.



As if the Circle, *A B C*, be circumscribed about the triangle. *A B C*, the side A B, is made the subtense of the angle, A C B, that is of the arch AB, which is opposite to the angle A C B.

The side B C, is made the subtense of the angle, B A C, that is of the arch B C, which is opposite to the angle B A C. And lastly, the side A C, is made the subtence of the angle ABC, that is of the arch A C, which is opposite to the angle A B C.

Therefore the side A B, is in proportion to the side B C, as the subtense of the angle A B C, to the subtence of the angle B A C, &c. Which was to be demonstrated.

The first Consectarie.

Therefore the angles being given, the reason of the sides is given; and consequently:

One side being given besides the angles, every of the other sides is given.

As in the plaine obliquangled triangle A B C, the angles being given at A, 20 deg. 10 min. at C, 60 deg. 13 min. and at B, 99 deg. 27 min. by the 3. Consectary of the 49 of the first, the reason of the sides is given after this manner.

A B. 8693512. the sine of the angle, A C B. 60. deg 23 min.

B C, 3447521. the sine of the angle, B A C. 20. deg. 10. m.

A C. 9864292. the sine of the angle, A B C, 99. d. 27. m. or the sine of the complement, to a semicircle being 80. deg. 33. min.

But

But if then, besides one of the sides bee giuen, (as for example) the side A B, 34. foot, the other sides shall also bee giuen, viz. B C, and A C. For

1 As A B. 8693512. is to B C. 3447521. So is A B. 34. foot to A C. 13. $\frac{7806053}{8693512}$. foot. And in like manner.

2 As AB, 8693512. is to A C, 9864293. So is A B. 34. foot to A C. 38. $\frac{3052475}{8693512}$. foot,

Or by changing the middlemost tearmes.

1 As A C B. 8693512. is to A B, 34. foot. So is B A C, 3447521. to B C, 13. $\frac{10}{17}$. foot.

2 As A C B. 8693512. is to A B, 34. foot : So is A B C, 9864293. to *A C*, 38. $\frac{12}{17}$. foot.

The second Consectary.

Two sides being giuen, with an angle opposite to the one of them the angle also opposite to the other of them is giuen.

As in the aforesaid obliquangled triangle, A B C. the two sides A B. 34. foot, and B C. 13. $\frac{7806053}{8693512}$. foot being giuen, with the angle, ACB. 60. deg. 23. m. opposite to one of the giuen sides, viz. to the side A B ; the angle B A C. opposite to the other of the giuen sides, to wit, to the side B C shall also be giuen.

For by the angle giuen, A C B, 60. deg. 23. min. the sine of angle *A B*, 8693512. is giuen.

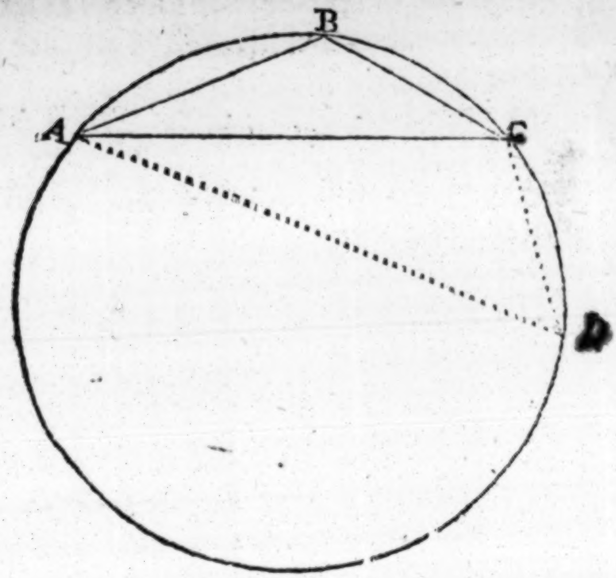
Then I say : As the side A B 34. foot, is to the side B C. 13. $\frac{7806053}{8693512}$. foot. So is A B, the sine of the angle, A C B, 8693512. to B C, 3447521. the sine of the angle A B C.

Or the middle tearmes being changed.

As A B, 34. foot to A C B, 8693512. So is B C, 13. $\frac{7806053}{8693512}$. foot, to *B A C*, 3447521. To which sine in the left Margent of the table, the angle of 20. deg. 10. min. answereth. Therefore the angle *B A C*, is 20. degr. 10. min.

Note. *In the vse of this consectarie, there may a doubt happen : that is to say If two sides be given whereof one of them is the greatest side together with the angle opposite to the lesser of the two giuen sides And the angle opposite to the greater of the giuen sides be demanded, For because that angle may be either acute or obtuse : and the sine to both of them, is the same by the 1. Con. of the 12. Pro. of the second.*

The doubt is, when you haue found the sine of the angle demanded whether that sine sheweth an acute, or an obtuse angle.



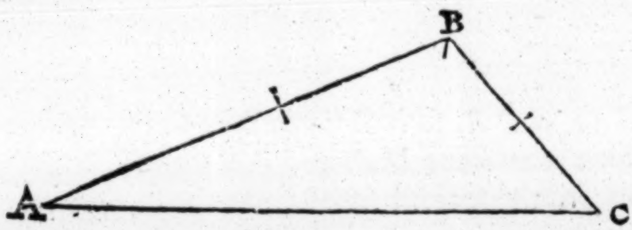
As in the obliquangled triangle A B C; If the two sides A C, 22, foote, and B C, 10. foote, together with the angle B A C, 24, deg. 50. min, 10. sec: And the angle A B C, opposite to the greatest side, A C, be demanded, If I shall say; As the side B C, 10. foot, to the side A C, 22. foot; so is B C, 4200241. the sine of the angle B A C, to A C, the sine of the angle A B C I shall readily find 9240530. to be the sine A C, But because that sine is the sine both of the acute angle A D C, 67. deg. 31. min. 34. sec. which the arch A B C, is opposite unto; and of the obtuse angle A B C, 112. deg. 28. min 26. sec. which the arch A D C, is opposite vnto : It is doubtfull whether the angle shewed by that sine be the obtuse angle 112. deg. 28. min. 26. sec. or the acute angle 67 deg. 31. m. 34. sec. Nor can this doubt be otherwise taken away, but that besides the other three things given, (which may be as well in the acute angled triangle A D C, as in the obtuse angled triangle ABC, For that the sides BC, and DC, and also the angles B A C, and D A C, are equall) this also be giuen, whether the angle sought for, be obtuse or acute : Or the same may be perceived by the true delineation of the triangle to bee resolued, whether the angle sought for be acute or obtuse.

The

The third Axiome.

In all plaine triangles: As the somme of two sides is to their difference; So is the Tangent *of halfe the somme of the two angles opposite to the* Tangent *of the difference, lesse or more then the halfe.*

The declaration : In the plaine obliquanled triangle, A B C, I say the *Tangent* of ½. the somme of the two angles at A, and C, is to the *Tangent* of the difference of the angle C more, and of the angle A, lesse then the halfe: As the somme of the two sides B C, and A B, opposite to those angles, is to the difference of those sides.



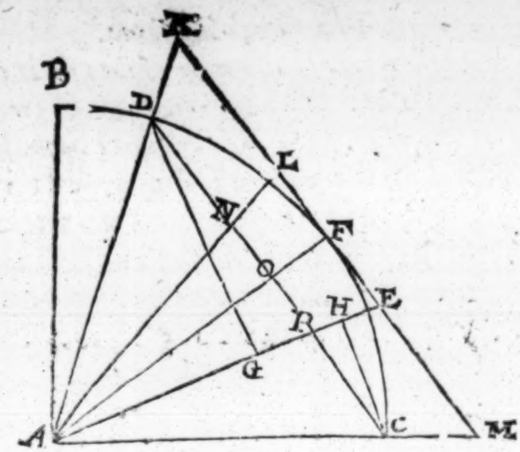
The demonstration. For that Quadrant A B C, being described make the angles D A E, and E A C, equall to the angles A C B, and B A, in the former scheme.

And therupon let the angle D A C, be the somme of those two angles, let the halfe so that somme be D A F, or F A C.

The difference of the angle, D A E, abone the halfe D A F, or of the angle E A C, lesse then the halfe, F A C, let be the angle F A E; let the subtense of the somme of the two angles, be the right line D C, Let the sine of the greater angle D A E, be the right line *D G*. Let the sine of the lesser angle *E A C*, be the right line *C H*. Let F M, or F K, be the *Tangent* of halfe the somme of the two angles: Let F E, be the *Tangent* of the difference, lesse or more then the halfe. Now the Triangles G D P, and HCP: are equiangled by the 4. Con: of the 49. of the 1. because of the equall angles D P G, and C P H, by the 13. of the first and the right angles at G, and H, by the 3. Con: of the 12. Pro: of the first.

Therefore, in this second scheme: As P D, to *D G*; so is P C, to *C H*, by the 46, of the 1. As in the first,

As



As AB. to *A C B.* so is B C. to B A C. by the second Axiom.

Therefore the sides *D P.* and *P C* haue altogether the same proportion in the second scheme; as the sides A B. and B C. in the first scheme.

Wherefore, putting the right line *D C*, for the summe of the two giuen sides A B, and B C, you may take the parts D P, and P C, for the same two sides, *A* B, and B C, by which position N P, is the difference of the two sides. But there the sides *A* B, and BC, here DP, and PC, are giuen; Therefore the difference also of their sides, NP, and the halfe thereof, O P, is giuen. Moreouer, for that the composed Triangles, *A* K L F E M, and *A D N O P C*, are euery where equiangled, because of the paralels D C, and K M; Therefore the sides and the segments of their sides, are proportionall by the 46. and 47. of the first. And thereupon.

As DC, the summe of the two sides, is to PN, their difference, so is K M, the double *Tangent* of half the summe of the two angles, to L E, the double *Tangent* of the difference, lesse or more then the halfe. Or.

As O C, halfe the summe of the two sides, is to OP, halfe their diffetence, so is F M, the *Tantent* of halfe the summe of the two opposite angles, to F E, the *Tangent* of the difference, lesse or more then the halfe, Or.

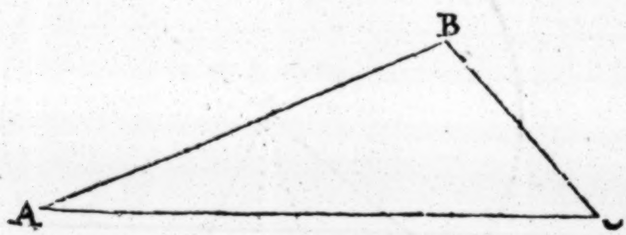
Retayning

Retaining the former two intire tearmes of the proportion and taking the halfe of the latter, you may worke more brieflie.

As D C, the summe of the two sides, is to N P, their difference, so is F M, the *Tangent* of halfe the two opposite aygles, to F E, the *Tangent* of the difference, lesse or more then the halfe. For as the whole to the whole. so is the part to the part. Therefore as the whole K M, to the whole L E, so is the halfe F M, to the halfe F E.

Consectarie.

Therefore in a plaine obliquiangled triangle, the two sides, being giuen, with the angle comprehended by them, the other two angles are also giuen.



As in the plaine obliquiangled triangle A B C, the two sides A B, *6.* and B C, 3, foote, with the angle A B G, being giue. 107. deg. 30. min. The angles *B A C*, and B C A, shall likewise bee giuen after this manner:

The summe of the two sides giuen is 9, their differenee is 3.

The summe of the angles at A, and C, is 72. degrees 30. min. by the 49. of the first.

The ½ thereof is 35. deg. 15. min. whose *Tangent is* 7332303.

Then I say.

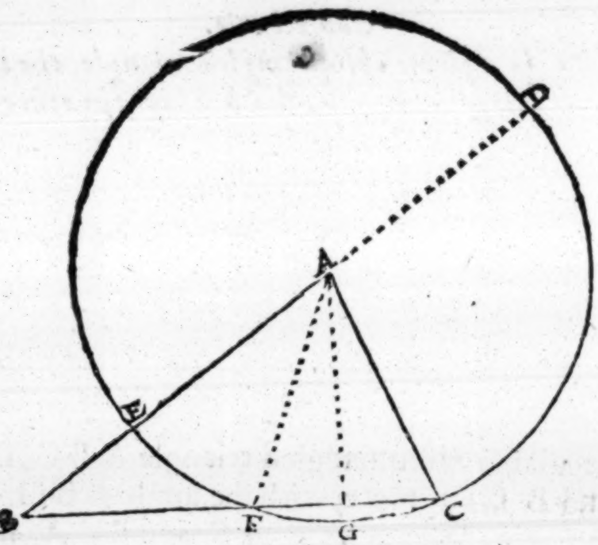
As the summe of the sides giuen 9. is to their difference 3. So is 7332303. the *Tangent* of halfe the summe of the opposite angles; to 2444101. the *Tangent* of the arch of 13. deg. 44. min. 4. sec. being the difference of the angle A, lesse, and of C, more, then the halfe. Therefore.

From 36. deg. 15. min 00 sec.				To 36. deg. 15 min. 00. sec.
Subtr. 13.	44.	4.		Add 13. 44. 04.
Rad, ÿ angle B A C, 22, 30. 56.				The whole is ÿ angle BCA (49. d. 59. m. 4. sec.

The

The fourth Axiome.

In all plaine Triangles. As the greatest side is the summe of the other sides; So is the Difference of those other sides, to the segment of the greatest side: Which segment subtracted, a perpendiculer shall fall in halfe the remainder.



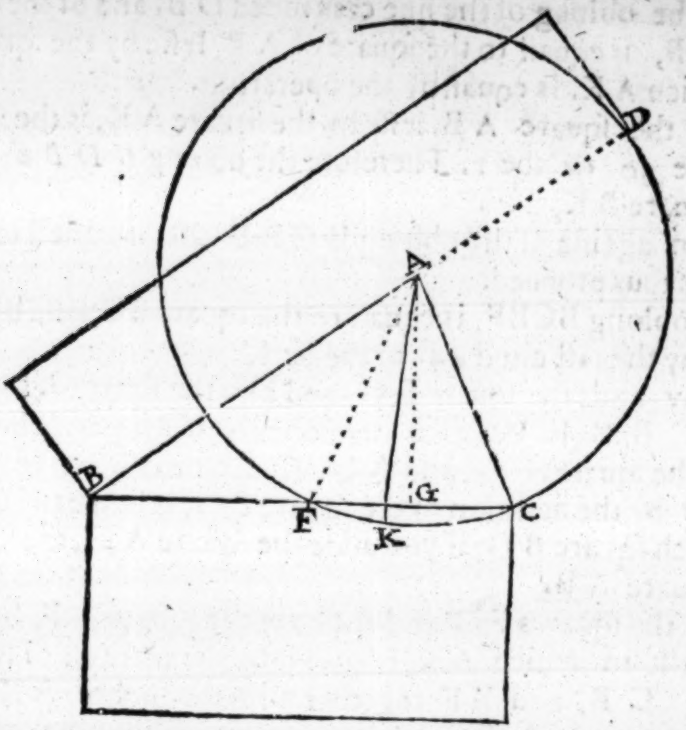
The Declaration. Let A B C, be an obliquangled triangle, and let the least side thereof, be A C, the greatest B C. At the distance of the least side *A C. A* being the center, let the circle *C D E F,* be described, cutting th'other two sides in the points E and F. Moreouer let the side *A B,* bee produced vnto D, and D B, shall bee the summe of the sides *A B,* and *A C.* For *A C,* and *A D,* are equall by the worke, then B E, shall bee the difference of the sides *A B,* and *B C*; for *A E,* and *A C,* are equall likewise by the worke.

I say first, that.

As *C B,* to *B D*; so is E B, to B F. Then that the prependiculer *A G,* doth bisect the right line F C.

The Demonstration. For as to the first, equall right angled figures haue their sides, reciprocally proportionall by the 2. Cons: of the 42. of the first.

But



But the oblongs made of B D and B E, and also of B C, and B F, are equall right angled figures.

Therefore they have their sides proportionall, reciprocally.

So that as B C, to B D;

So is B E, to B F.

The Minor is proved. For whatsoever is equall to one and the same thing, are equall to one another. But the Oblongs made of B C, and B F; and also of B D, and B E, are equall to a square made of the right line B K, being the Tangent of the angle BAK. Therefore also they are equall one to another.

Againe, the Minor is proved. And first of the Oblong B D, and B E; that is equall to the Square B K, is thus proved. If a right line bysected bee continued, the Oblong made of the line continued, and the Continuation is equall to the Square, made of the right line composed of the bysegment, and the Continuation, lesse by the square of the bysegment; by the 44 of the 1. But E D, is a right line bysected in A, and continued from E to B. There-

fore

fore the oblong of the line continued D B. and of the continuation E B, is equall to the square of A B, lesse by the square of E A, to which A K. is equall by the operation.

But the square A B, lesse by the square A K, is the square B K. by the 50. of the 1. Therefore the oblong *B D B E*, is equall to the square B K,

Then againe of tho oblong B C B F. that is equall to the square B K is thus prooued.

The oblong BCBF, is equall to the square BG, lesse by the square F G. by the last cited 44. of the first.

Now adde the square F G, and also the square A G, to the oblong C B B F. Which being done the oblong B C B F, together with the squares F G, and A G. shall be equall to the square A B,

For by the addition of the square F G, is made the square B G. to which square B G, if you adde the square A G, thereof is made the square A B.

But the squares F G, and A G, are the square A F, by the 50. of the first, to which A K, is equall by the worke; Therefore the oblong C B, and B F, together with the square, A K. is equall to the square, A B. And thereupon without the square, *A K*, is equall to the square *A B*, lesse by the square, *A F*, that is to the square B K. by the 50. of the first.

But that which I propounded in the second place, of the perpendiculer A G, bisecting the right line *C F*, it is thus prooued.

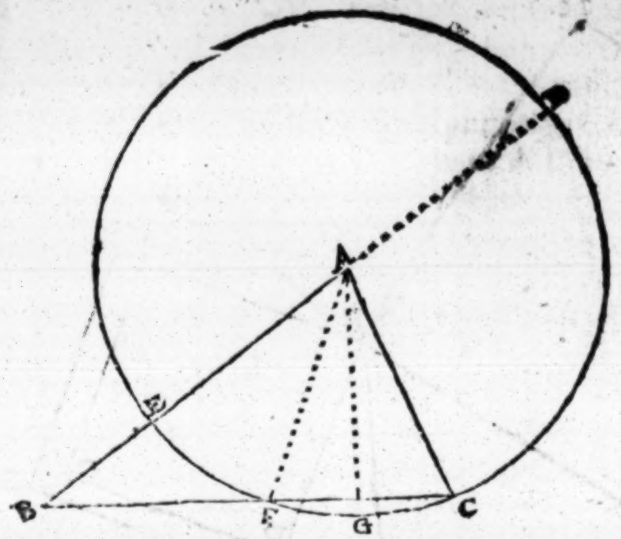
Because the Triangle F A C, is of two equall sides, in *F A*, and *A C*, By the worke. Therefore the perpendiculer A G, bysecteth the base F G, by the 23. of the first.

Therefore in an obliquangled Triangle; As the greatest side to the summe of the other sides, so is the difference of those other sides, to the segment of the greatest side, which taken away, the perpendiculer, shall fall in ½ the Remainder, which was to bee demonstrated.

Consectarie.

Therefore the three sides of a plaine obliquangled Triangle, being giuen, whether segment you will of the greatest side is giuen, in which from the greatest Angle, the perpendiculer shall fall.

As



As in the plaine obliquiangled Triangle, ABC, Let the 3 ſides be given.

A B. 22. Foot.
B C. 21. Foot.
A C. 13. Foot.

And let the ſegments of the greateſt ſide B C, in whoſe concourſe the perpendiculer ſhall fall, to wit, the right line B G, and G C, be demanded.

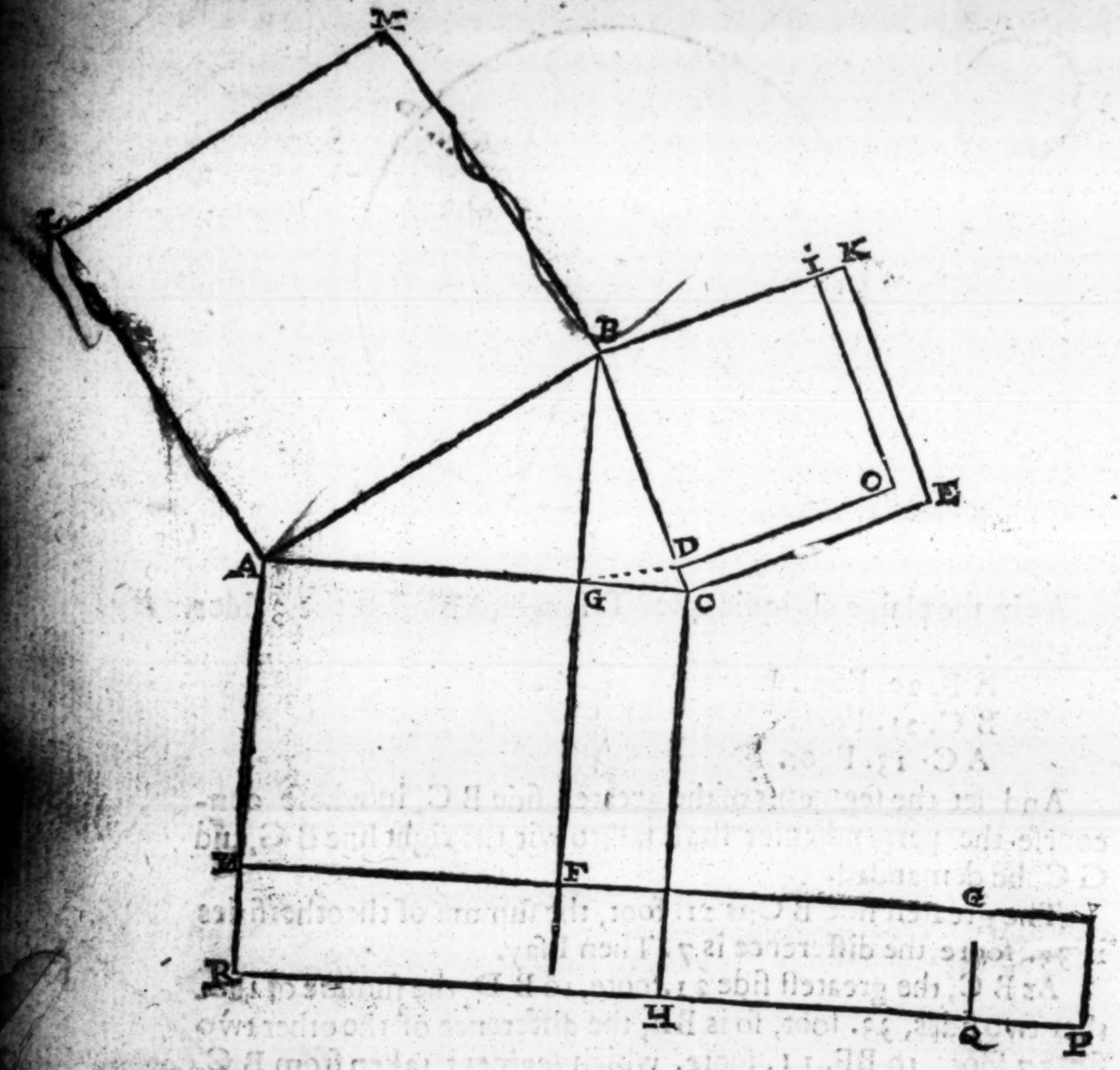
The greateſt ſide B C, is 21. foot, the ſumme of the other ſides is 33. foote, the difference is 7. Then I ſay.

As B C, the greateſt ſide 21. foote, to B D, the ſumme of the other two ſides, 33. foot, ſo is BE, the difference of the other two ſides 7 foot, to BF, 11. foote. which ſegment taken from B C, 21. foot, the Remainder is FC, 10. foot, whoſe ½. is FG, or GC, 5. foot. Therefore G C, is 5. foot, and G B. 16. foot.

An Annotation.

The fourth Axiome may alſo be thus propounded.

From the ſumme of the ſquares of the baſe and of one of the ſides, ſubtract the ſquare of the other ſide, the remainder divide by the baſe doubled, and you ſhall have the ſegment of the baſe interjacent, or lying betwixt the perpendiculer, and the ſide firſt taken.



The Declaration. Let A B C, be an obliquiangled Triangle, of three unequall sides, and let the given sides be A B, B C, and A C.

And let the segment G C, betwixt the perpendiculer B G and the side B C, interjacent be demanded. I say, if from the summe of the squares of the base A C, and of the side B C, the square of the side A B be subtracted, and the Remainder divided by the base A C doubled, the quotient shall be the segment G C.

The

The Demonstration. For the square of the side A B, to wit, the square A L M B, is equall to the squares of the sides A G, and B G, added together, by the 50. of the first.

Now the square of the side A G, is A N F G, and the square of the side B G is B D O I, making the right lines B G, and B D, equall. Therefore if you subtract the square A L M B, from the summe of the squares A R H C, and BCEK, there shall remain the two Gnomons N H G, and D E I.

But the gnomon D E I, is equall to the square of the side G C. For the square of the side B C, is equall also to the square of the sides of G B and G C, by the 50 of the first. But the square of the side B G, is now taken from the square of the side B C; Therefore that, that remaineth, is equall to the square of the side G C, which square if you adde in the right line R H Q, extended to the gnomon NHC, you shall haue the oblong N C P R, which diuided by the length N C, that is by the double base A C, the quotient shall bee the bredth C P, equall to C G, by the worke.

The illustration by Numbers. Let the side be giuen as before *A B*, 20. B C, 13 and A C, 21. foot. And let it bee demanded how many foote is the segment G C?

Answer 5. The whole worke shall be thus.

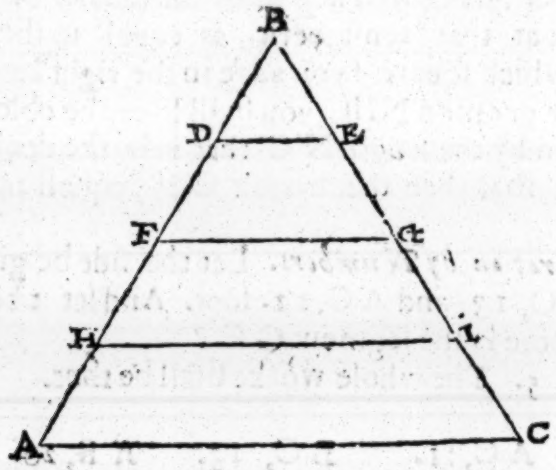
A C, 21.	B C, 13.	A B, 20.
21.	13.	20.
21,	39.	q. 400.
42.	13.	
The square 441.	q. 169.	
The square 169.		
The totall. 610.		
Subtract. 400.		

The Remainder, 210. Which diuided by 42. the double of A C, the quotient is 5. for G C.

The vse of the precedent Axiomes, Or :

A Manuduction, wherein is shewed, how by the helpe of a few of those Axiomes, every plaine Triangle may be resolued.

In euery Triangle there are 6. termes, to wit 3. sides and 3. angles: of these whatsoeuer 3 bee giuen in a plaine Triangle, the other 3, may be found out by the 4. Axiomes afore-going, and sometimes diuers wayes, onely one case excepted: that is, if the 3. angles be onely giuen; for thereby no side can bee found out: Because the 3 angles of one Triangle may bee equall to the 3 angles of another Triangle, although their sides be altogether vnequall.



As the three angles of the Triangles AB C, and D B E are equall, for that their bases A C, and D E, are parralell by the 38. of the first: And yet the sides of the Triangle A B C, are farre greater then the sides of the Triangle D B E. Therefore this case onely is excepted in Trigonometria: In all other by whatsoeuer three termes giuen, euery fourth may be found out, which by laying downe all the cases, I will demonstrate after this manner.

A plaine Triangle is right angled or oblique-angled.

1 *In a right angled plaine Triangle either all the angles (that is, one of the acute angles being giuen) with one side are giuen, and the other two sides are demanded.*

2 Or

2 Or else two sides with one angle, that is; the right angle are giuen: and the other two angles with the third side, are demanded. In both which cases, the first Axiom is sufficient.

In a plaine oblique angled Triangle.

1 *Either all the angles are giuen* (For, as often as two are giuen, the third is alwaies the complement of the other two, to two right angles by the third Consi: of the 49. of the first,) *with one side, and the other two sides are demanded.*

2 *Or two sides with one angle opposite to the one of the giuen sides are giuen: And the angle opposite to the other of the giuen sides, together with the third side, is demanded.*

3 *Or two sides with an angle comprehended by them are giuen: And the other two angles with the third side are demanded.*

4 *Or lastly, all the three sides are giuen: And the angles are demanded.*

The first Axiom is fully sufficient for the first two cases.

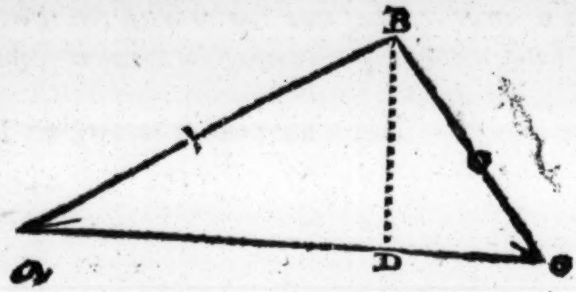
2 In the third case, the two vnknowne angles are found out by the third Axiom: And then the third side by the second Axiom.

In the fourth case first diuide the plaine obliquangled Triangle into two right angled Triangles, by letting fall a perpendiculer vpon the greatest side, by the fourth Axiom; Then in these right angled Triangles, euery angle is found out by the first Axiom.

But the former three cases may bee performed by the first Axiom onely. So as a perpendiculer be let fall from any angle vnknowne, vpon any of the opposite sides vnknowne, either within or without the Triangle (in which case the sides vnknowne is to be increased somtimes) and so the plaine oblique angled triangle will be diuided into two right angled triangles, whether the perpendiculer fall within or without the Triangle. And yet this rule (that the side vpon which the perpendiculer is to fall, ought to be vnknowne) is onely meant in the examples of the second Axiom, and not in the examples of the third Axiom of plaine Triangles.

1 As if such a porportion bee giuen.

As *A B* the sine, of the angle *A C* B, to B C, the sine of the angle B A C, So is the side A B, to the side BC, by the second Axiom.

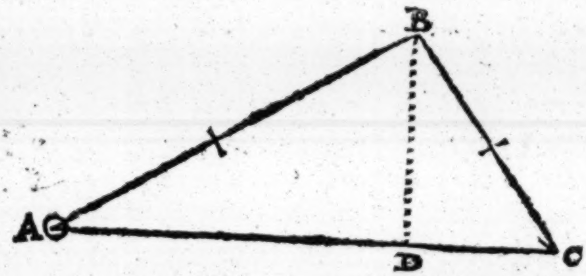


With the same effect you may say by the first Axiome.

1 As A B, the Radius, to B D, the sine of the angle B *A D*, So is the side *A* B, to the side B D.

2 As B D, the Radius to B C, the Secant of the angle *D* B C, (which is the complement of the angle B C D, So is the side B D, to the side B C.

2 If such a proportion bee giuen, As the side AB, to the side B C, So is A B, the sine of the angle A C B to B C, the sine of the angle B A C by the second Axiome.

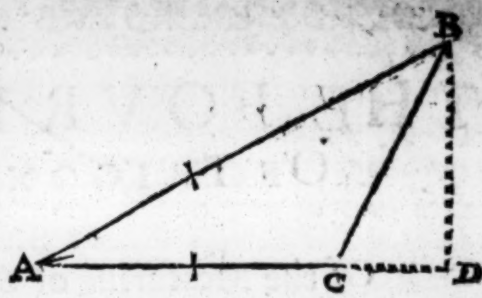


With the same effect you may say by the first Axiome.

1 As B C, the Radius to B D, the sine of the angle BCD, So is the side B C, to the side B D.

2 As the side BD, to the side AB, So is BD, the Radius to A B, the Secant of the angle *A* B *D*, whose complement is the angle *B A D*.

And that angle *A* B *D*, added to the angle DBC, maketh the angle *A B C*.



3 If such a porportion were giuen, As the summe of the sides A B, and A C, to their difference: So is the Tangent of halfe the summe of the angles A B C, and A C B, to the Tangent of the difference more or lesse then the halfe, by the third Axiome.

With the same effect you may say, by the first Axiome.

1 As *A* B, the Radius to B D, the sine of the angle *A* B*D*. So is the side *A B*, to the side B D.

2 As A B. the Radius to *A D*, the sine of the angle A B D, So is the side A B, to the side A D. From whence, if you subtract the side A C, there resteth the side D C,

3 As the side C D, to the side D B, So is the Radius D C, to D B, the tangent of the angle D C B, which added to the angle B A C, and the totall subtracted from two right angles, the remainder will be the angle *A B C*.

But if also you would find the side B C, you shall likewise say by the first Axiome.

As D C, the Radius to B C, the secant of the angle D C B, So is the side D C, to the side B C.

The end of the third Booke.

THE FOVRTH BOOKE OF TRIGONOMETRIA.

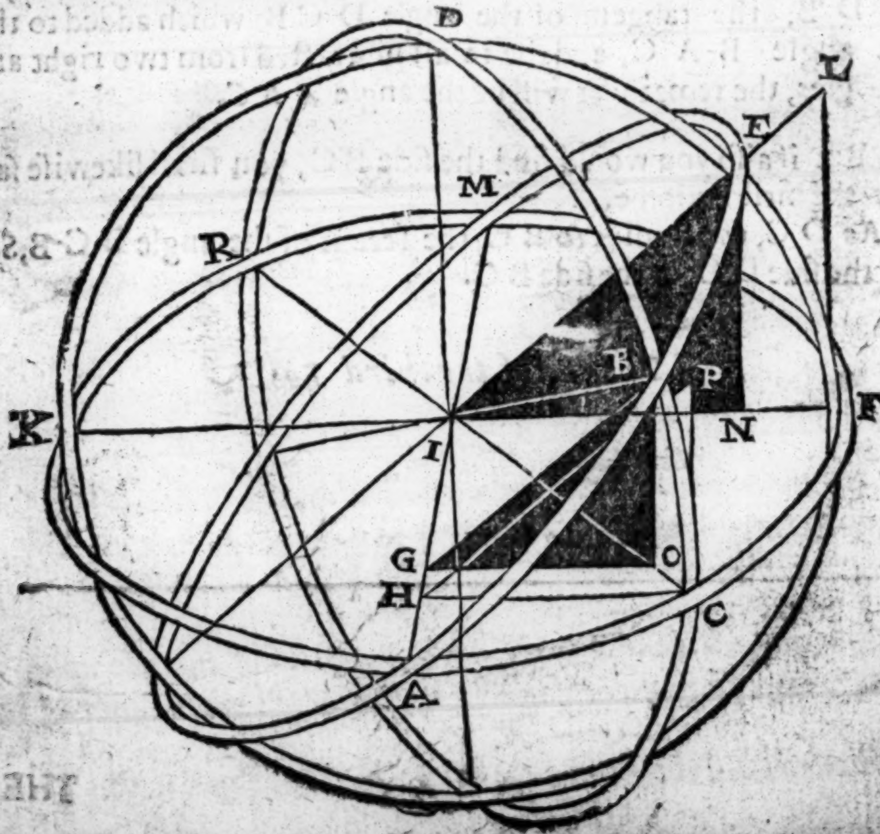
By B. P.

Of the Measuring of Sphæricall Triangles.

THE chiefe Axiomes of proportions that are in Sphæricall Triangles, and altogether sufficient for their resolution, are foure.

The first Axiome.

In many right angled Sphæricall Triangles, having one and the same acute angle at the base, the sines of the Hypothenusaes, and of the perpendiculers, are all of them proportionall one to another.



The Declaration.

Let K M F A D, be a Sphære given, and therein let K M F A, be the horizon, the pole of the Horizon D. Let K D F, and R D C be circles passing by the Pole of the horizon D, and cutting the horizon at right angles in K R F, and C, by the 57 of the first. Let M E A, be an oblique circle to the Horizon, cutting the verticall circle K D F, at right angles at E, for that it passeth by the poles thereof M and A, by the 57 of the first. And in like manner is cut by it into two quadrants M E, and E A, by the 56 of the first. In this Spheare, and in this position of circles, there are amongst other two right angled Spearicall triangles A B C, and A E F. And in them let the hypothenuses be A E, and A B, the perpendiculars E F, and B C, the bases A F, and A C, and the acute angle at the bases A F, and A C, let be the same angle E A F, or B A C. Lastly, let the sines of the hypothenuses A E, and A B, be the right lines I E, *Radius* and G B. And the sines of the perpendiculers E F, and B C. Let be the right lines E N, and B O, by the 12. th of the second booke. Now I say, that those sines of the hypothenusaes, and of the perpendiculers, to wit, the sines of I E, G B, E N, and B O, are all proportionall one to another. And so any three of them being given, the fourth may be found out. More plainely I say, that

As I E to E N, so is G B to B O. And in like manner,
As G B to B O, so is I E to E N. And contrarily,
As N E to I E, so is O B to B G.

Or by changing the middle termes by the 42 of the first.
As I E to G B, so is E N to B O. And in like manner,
As G B to I E, so is B O to E N. And contrarily,
As N E to O B, so is I E to B G.

The Demonstration.

For if you joyne together the sines G B and B O, by the right line G O; that thereby may be made the Triangle G B O, it is manifest that the Triangles G B O, and I E N, are equiangled. For first, because the right lines E N, and B O, fall perpendicularly upon the subjected plaine M F C, by the supposition, and by the 3. Con: of the 12. of the 2. Therefore they make right angles with all the lines drawne in the same plaine, and so the angles E N I, and B O G, are right angles. Then because the right line

I E, and G B, are paralell one to another, by the 38 of the first, for they are drawne alike upon the same right line I A, by the 3. Con: of the 12 of the 2. And because the whole plaine M E A, is every where inclined with the same angle to the plaine M F A, therefore also the paralels drawne therein I E, and G B, are inclined with the same angles to the paralels I N, and G O, under them in the plaine M E A, and so the angles E N, and G B O, are equall. And consequently in the Triangles I E N, and G B O, where two angles are equall to two; There also the third is equall to the third by the 49 of the first; And thereupon the Triangles I E N, and G B O, are equiangled. But if they be equiangled, they have the sides about the equall angles proportionall by the 46 of the 1. And so they are,

As *I E* to *E N*. So is G B to B O, &c. which was to be demonstrated.

The illustration by Numbers. *Then let the hypothenusaes A E 90 deg. and A B 42 deg. together with the perpendiculer* E F, 48. *deg.* 25 *min. be given: And let the perpendiculer* B C, *be sought for.*

Of the arches given —	A E, 90 deg. / A B, 42. — / E F, 48.d. 25.m	the sines are	I F, 10000000. / G B, *6691306.* / E N, 7479912.

Then I say, I E, 10000000. to E N, 7479912. So is G B, *6691306.* to B O, 5005038. But the arch of 30 deg. 2. min. in the Tables, answereth to the sine 5005038. Therefore the perpendiculer B C, is 30 deg. 2 min.

In like manner. Let both the hipothenusaes with their sines be given as before: But of the perpendiculers, let the perpendiculer B C, 30. *deg. two min. bee now given together with his sine B O*, 5005038. *And let the perpendiculer E F, be sought for: I say:*

As *G* B, *6691306.* to B O, 5005038, So is *I E*, 10000000. to E N, 7479912. But the arch of 48 deg. 25. min. in the tables, answereth to the sine 7479912. Therefore the arch E F, is 48. degrees 25. minutes.

Contrarily. Let both the perpendiculers E F, and B C; *together with the greater hipothenusa A E, be given, And let the lesser hipothenusa* A B, *bee sought for. I say:* As *E N*, 7479912. to *I E*, 10000000. So is B O, 5005038. to *G* B, *6691306.* But the arch of 42. degrees in the tables, answereth to the sine *6691306.*

Therefore

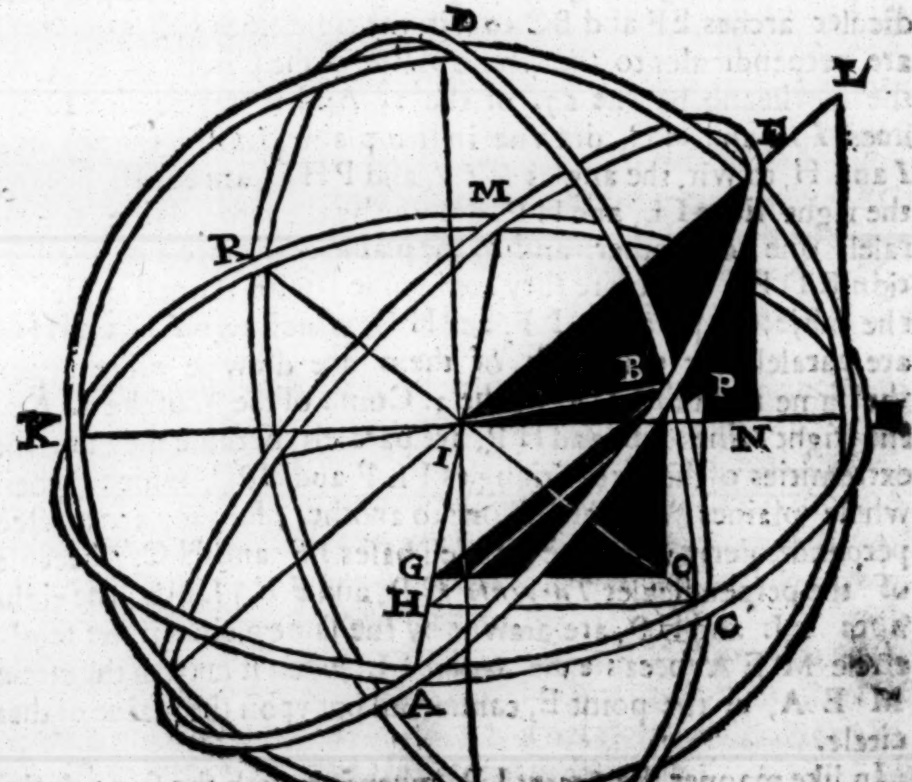
Therfore the hipothennsa *A B*, is 42. degrees

The second Axiome.

In many right angled sphæricall Triangles, hauing the same acute angle at the base: The sines of the bases and the *Tangents* of the perpendiculers are all porportionall one to another.

The declaration: In tae former diagram, and in the same Triangles *A E F*, and A B C, wherein the sines of the bases A *F*. and *A C*. are *I F*, and *H* C, but the *Tangents* of the perpendieulers, E F, and B C, are L F, and P C.

I say that those sines of the bases, and the *Tangents* of the perpendiculer, that is the sines *I F*, and H C, and the *Tangents L F*, and P C, are all of them proportionall one to another. And so, any three of them being giuen, the fourth may be found out. More plainely; I say that.



As *I F*, to F L, So is H C, to P C, And in like manner.
As H C, to C P, So is *I F*, to F L. And contrarily,
As F L, to *F I*, So is P C, to C H.

Or by changing the middle termes, by the 42. of the first.
As *I F*, to H C, So is F L, to C P, And in like manner,
As H C, to *I F*, So is C P, to F L, And contrarily,
As F L, to P C, So is *F I*, to C H.

The Demonstration. For when you haue drawne the right lines *I L*, and H P, and accomplished the Triangles *I L F*. and H P C, those Triangles *I L F* and H P C, shall bee equiangled, and therefore proportionall in their sides, by the 46. of the first. And the triangles *I L F*, and H P C, shall be equiangled because of the right angles, at F, and C, and the equall angles at *I* and H, and thereupon also at L. and P, by the 49. of the first. Moreouer, the angles at F, and C, to wit, the angles *I F L*, and H C P, ar right angles, because the *Tangents* of the perpendiculer arches, EF, and BC, to wit, the right lines L F, and P C. are perpendiculer to the whole plaine of the Circle, M F A, by the worke and by the 17. of the 2. And therefore also to the lines *I F*, and H C, drawne in that plaine. Lastly, the angles at *I* and H, to wit, the angles *L I F*, and P H C, are equall, because the right lines I L, and H P, drawne by the same plaine, are paralell one to another, and to the plaine of the circle of inclination K D F; Therefore they are inclined with equall angles, to the subjected paralels I F, and H C, which two *I F*, and H C, are paralels for that both of them are drawne alike vpon the same right line *I A*, by the 3. Cons: of the 7. of the 2. And the right lines *I L*, and H P, are paralels, because they are the extremities of the two Triangles I L F, and HPC, which in their whole plaines are paralell one to another; for they are erected perpendicularly vpon the paralell bases I F and H C, (because of the perpendiculer *Tangents* C P, and F L.) Lastly, the right lines I L, and H P, are drawne by the same plaine of the semicircle M E A: because the *Secant* I L, when it cutteth the circle M E A, in the point E, cannot fall but vpon the plaine of that circle.

In like manner, the *Secant* I P, when it cutteth the same circle, *M E A*, in the point B, cannot fall but vpon the plaine of the same

same circle: Which plaine, because it is a plaine, if it should bee extended according to the right line I P, it should fall vpon the *Tangent* P C, in the point P. And so the point P, should be in the plaine of the Circle M E A, so extended. But in the same plaine is the point H, appointed. Therefore the right line P H, is a line falling betwixt two points of the said plaine, and there upon drawn by the same plaine. All which was to be demonstrated.

The illustration by Numbers, *Let therefore the two bases A F, 90, deg. A C, 30 deg. 51. min. 46. sec. Together with the perpendiculer E F, 48. deg. 25 min. be giuen. And let the perpendiculer* B C, *bee sought for.*

Of the bases { AF 90. deg. ——— AC. 30 d. 51. m, 46. s. } The sines are { 10000000. IF. 5129838. HC.

of the perpendiculer E F, 48 d, 25 m, ye *Tangent* is 11269872, L F. Then I say.

As I F. 10000000, to L F, 11269871. So is H C, 5129838. to P C, the *Tangent* 5781262.

But to the *Tangent* 5781262. in the tables, the arch of 30 deg 2. min answereth. Therefore the perpendiculer B C, is 30 d, 2. m.

In like manner, let the two bases together with their sines be giuen as before: But of the perpendiculers, let now the perpendiculer B C, *30. deg. two min. together with his* Tangent *C P*, 5781262. *bee giuen, And let the perpendiculer E F be sought for.* I say: As H C, 5129838. to C P, 5781262. So is I F, 10000000. to F L, the *Tangent* 11269872.

But the arch of 48. deg. 25. min. in the tables, answereth to the *Tangent* 11269872. Therefore the perpendiculer E F, is 48 degrees 25. min.

Contrarily; let both the perpendiculers E F, and B C, and their Tangents *L E, and P C, together with the greater base A F, and the sine thereof I F, be giuen: And let the lesser base A C, or rather the sine thereof H C, be sought for: I say.*

As L F, the *Tangent* 11269872. to F I, the *Radius* 10000000. So is P C, the *Tangent* 5781262. to H C, 5129838. the sine of the arch of 30. deg. 51. min. 46. sec. Therefore, the arch or the base A C, is 30 degrees 51. minutes 46. seconds.

The Appendix, By these two Axiomes and their declarations and demonstrations, the ingenious reader may vnderstand, why

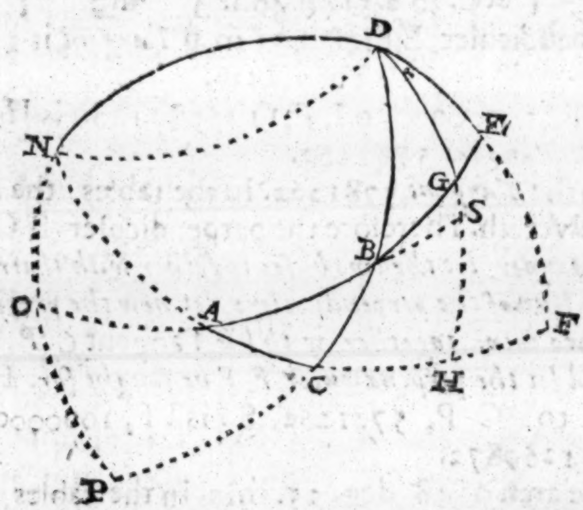
there

there is no reason or proportion, betwixt the sines of the bases and the sines of the perpendiculers, & contrarily. When notwithstanding there is proportion betwixt the sines of the hipothenuses and the sines of the perpendiculers and contrarily; to wit, because the sines of the bases, and of the perpendiculars doe not meet together in the same right lined Triangles. Which thing also you may see that some Mathematicians otherwise very learned haue sometime pretermitted.

3

The third Axiome.

In all Sphericall Triangles, the sines of their sides are directly proportionall to the sines of their opposite angles.



The Declaration. First, let ABC, be a Sphæricall triangle right angled at C. Then let the sides A B, A C and C B, be continued, to make the Quadrants A E, A F, and C D. and from the Pole of the quadrant A F, to wit, from the point D, let be drawne downe two other quadrants D F, and D H. And so is made three new triangles, that is the right angled triangles *B D E*, and *G D E*, and the obliquiangled triangle B D G, J say in the right angled Sphæricall triangle A B C.

As A C B, to A B. So is A B C, to A C; and so is B A C, to B C.

Or by changing the middle tearme by the 42 of the first.

As A C B, to A B C, so is A B, to A C, And,

As A C B, to B AC, so is AB, to B C, &c. Likewise in the obliquiangled sphæricall triangle B D G, I say, that

As B D G, to B G so is B G D, to B D, and so is D B G, to D G &c.

The demonstration For as to the right angled triangle ABC. In it A C B, and A E, also B A C, and E F, and on the other side A B C, and O P, that is the angles: and the measure of those angles. are of the same quantity.

(For as E F, is the measure of the angle E A F, and O P, the measure of the angle ABC, so is ND, or AE, or OB (equall thereunto) the measure of the angle ACB, by the 57. of the 1.)

Therefore it is all one, If J shall say.

As A C B, to A B, So is B A C, to B C, or,

As A E, to A B, So is E F, to B C. But this is proved by the first Axiom of sphæricall triangles; and therefore that also.

In like sort it is all one, as if I shall say,

As *A C B*, to *A B*, So is *A B C*, to *A C*, or,

As O B to *AB*, So is O P, to *A C*. But this is proved by the first Axiom of sphæricall Triangles, and therefore that also. For those things that are agreeable to a third, are agreeable one to another. But by the Rules demonstrated.

As *A C* B, to A B, So is A B C, to *A C*, And so is B A C, to B C, Therefore also,

As A B C, to A C, So is B A C, to B C.

Then as to the obliquiangled Triangle BDG, Because by the demonstration of right angled Triangles, they are

As D B, to D E B, So is D E, to *D B E*. And

As D G to D E G. So is D E, to D G E, or (by the first Con: of the 12, of the 2.) to D G B, Therefore changing of the proportionall termes, it shall be

As D G to *D B*, So is *D B E* or *D B G*, to *D G B*, &c.

And likewise, if from the point B, a perpendiculer arch bee let fall to S. Because then

As B D, to *B S D*, So is B S, to *B D S*. And

As B G, to B *S G*, So is B S. to *B G S*, Or by the first Const of the 12, of the second to *B G D*; Therefore also

As *B G*, to *B D*. So is B*DS*, or B*DG*, to *D G* B, &c. For if

As

As 4. to 12. So is 1. to 3. And
As 2. to 12. So is 1. to 6. Then
As 2. to 4. So is 3. to 6.

The illustration by Numbers. Then in the right angled Sphericall Triangle A B C. First, let A C B, A B, and A B C, bee giuen, in the same quantity as before: And let the side A C, opposite to the angle A B C, be sought for. I say.

As *A C B* 90. deg. to *A B*, 42. deg. So is *A B C*. 50 d. 3. m. 12 s.

10000000. 6691306. 7666422.

to A C, 30. deg. 51. min. 46. sec.

5129838.

Or Contrarily. Let A B, and A C B, and A C, be given: And let A B C, bee sought for; I say,

As A B, 42. deg. to A C B, 90. deg. So is A C. 30, deg. 52. mi.

6691309. 10000000 5129838.

to ABC, 7666422. the sine of y^e arch or angle of 50. d. 3. m. 12. s.

Againe let A C B, A B, and B A C, be giuen, And let B C, *bee sought for. I say,*

As A C B 90. deg. to A B, 42. deg. So is B A C, 48. deg. 25. m.

10000000. 6691306. 7479912.

to BC, 5005038. the sine of the arch of 30. deg. 2. min.

Or contrarily. Let A B, *A C B, and* B C, *bee giuen. And let* B *A C, be sought for. I say,*

As A B. 42. deg. to A C B, 90. deg. So is B C, 30. deg. 2. min.

6691306. 10000000. 5005038.

to B A C, 7479912. the sine of 48. deg. 25. min.

Lastly, Let B A C, B C, *and* A B C, *be giuen: And let* A C, *be sought for. I say.*

As B A C, 48 d. 25. m. to BC. 30 d. m. So is ABC. 50 d. 3 m. 12 sec.

7479912. 5005038. 7666422,

to A C, 5129838. the sine of 30. deg. 51. min. 46. sec.

Or

Or Contrariwise. Let B C, *B A C, and A C, be given : And let A B C, be demanded. I say,*

As BC, 30. d. 2 m. to BAC, 48. d. 25 m. So is AC. 30. d. 51 m. 46. s.

5005038 — 7479912 — 5129838

to A B C, 7666422, the sine of 50. deg. 3. min. 12. sec.

In like manner, is the obliquangled Sphericall Triangle B D G: First, let D B G, DG, *and* BDG *be given, And let* B G, *bee sought for. I say,*

As D B G 50 deg. 3. m. 12. sec. to D G, 45 deg. 57 m. 41 sec. So is B D G, 28. deg. 14. min.

7666422 — 7188714 — 4730634

to B G, 4435860. the sine of 26. deg. 19. min. 58. sec.

Againe, Let B G, BDG, *and* D G, *bee given. And let* DBG *be sought for. I say,*

As B G, 26. deg. 19 min. 58. sec. to B D G, 28. d. 14. m. So is D G 45 deg. 57 min. 41. sec.

4435860 — 4730634 — 7188714

to DBG, 7666422, the sine of 50. d. 3. m. 12 sec.

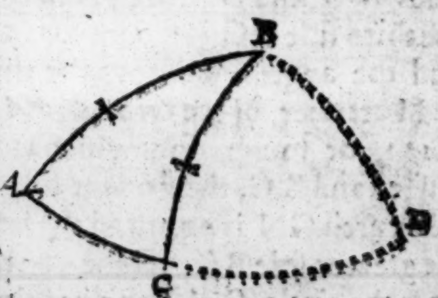
Lastly, Let D G, D B G, *and* D B, *be given : And let* D B G, *be demanded. I say,*

As D G, 45 d. 57. m. 41 sec. to D B G 50 deg. 3 m. 12 sec. So is D B. 39 deg. 58. sec.

7188714 — 7666422 — 8657344

to DGB, 9232491, the sine of the obtuse angle 112 d. 25 m. 40 s.

Note. *In the use of this Axiom the same doubt may fall out as I have formerly said, might happen in the use of the second Axiome of plaine Triangles : As appeareth by the like Scheme* A B C D. *Therefore it behoveth you to bee diligent, least in such case you be deceived in finding an acute Angle for an obtuse; Or contrarily,*



4 th

The fourth Axiom.

In all Sphæricall Triangles. If first you adde the two sides either of them lesse then a quadrant, together, and then adde the lesser side to the Complement of the greater side; And if you subtract the sine of the Complement of the former composed arch from the sine of the latter composed arch: Or if you adde it to the sine of the excesse. Then,

As the Radius is to that halfe of the right line, so made either by addition or substraction: So is the versed sine of the angle comprehended of the said two sides to a right line, which subtracted from the sine of the latter composed arch, leaueth the sine of the Complement of the third side: Or from whence the sine of the latter composed arch subtracted, leaueth the sine of the excesse of the third side.

Or Contrarily. As the halfe of that right line to the Radius: So is the right line made of the sine of the latter composed arch either by subtraction of the sine of the Complement of the third side, or by addition of the sine of excesse of the same third side, to the versed sine of the angle comprehended of the other two sides:

The Declaration. This Axiom hath diuers cases.

For first, the two sides including the angle giuen or sought for, added together, are either equall or vnequall to a quadrant, and that either lesse or more.

Then the angle giuen or sought is either right or oblique, and that either acute or obtuse.

Lastly, the third side opposite to the said angle, is lesse or more then a quadrant. All these cases, I shall plainly explaine as I think, in three schemes. In euery of which, the obliquangled Triangle, propounded is for examples sake A B C, wherein either the two sides A B, and B C, are giuen together with the angle at B, And the third side AC, is sought for; or else all the three sides are giuen, and the angle opposite to the third side *A C*, is sought for.

Moreouer, of the two sides *A B*, and *B C*, including the angle giuen, or sought for (which is alwaies placed at B,) A B, is the lesser, and B C, the greater side.

The arch G N, is equall to the lesser side AB, by the worke. From the greater side B C let the equall arches B F, and B D, be cut off from the circle *D AB* by a paralel, described on the superfices of ẏ Globe; B, being the pole, and BC, the distance of the compasse: the

Diameter of which parallel is DCF, the circumference, (only noted in the first scheme) D X F. the point X, meeting in the Globe with the point C, of the great circle BC, And in that same parallel D X F, let be noted the arch D X, for the measure of the angle at B, by the sixe of the first, and his right sine X C, by the 12, of the 2. and the versed sine D C, by the 13. of the second.

Lastly, let the equall arches A K, and A M, in like sort, bee cut off from the circle D A B, by the parallel *K C M*, described in the superficies of the Globe, A, being the pole, and the distance of the compasse *A C*.

These things being thus laid downe. First let the sides A *B*, and B C, or B F, including the angle A B C, either giuen or sought for, be added together. And let the former composed arch be A F, in the first scheme, equall to A *Q*, the quadrant: In the second lesser, and in the third more. And let V F, in the second scheme (being the sine of the Complement) and in the third (the sine of the excesse) bee noted, Then let the lesser side A B that is (by the worke) G N, bee added to the Complement of the greater side G D and let D N, bee the latter composed arch, and his right sine D P. From which sine D P, let the sine of the complement V F, or P R, (in the second scheme) bee subtracted: But in the third scheme, let the side of the excesse V F or P R, bee added to the sine D P, that thereby the right line D R may bee found, which ioyned with the rightline DF, by the right line R F, maketh the plaine rightangled Triangle DRF, by the halfe whereof let the right line T E, be drawne, cutting in halfe the right line D F, in E, by the worke and so also the right line D R, by the 45. of the first, making the Triangle D T E, equiangled to the Triangle D R F, by the 38. of the first, which Triangle D T E, thus made, I say that,

As the Radius E D, to the halfe of the right line D R, to wit, to the right line D T. So is the versed signe of the angle *A B C*, to wit, the right line D C. to the right line D L, which taken from the sine of the latter composed arch D P, there remaineth the right line L P, or K O, by the 39. of the 7. beeing the right sine of the arch K N, or C S, the Complement of the third side A C.

And Conversely.

As the halfe of the right line being D T, is to the Radius D E: So is the right line D L, (after the subtracting of the sine of the Complement of the third side from the other sine D P,) to the versed sine D C, &c.

The Demonstration.

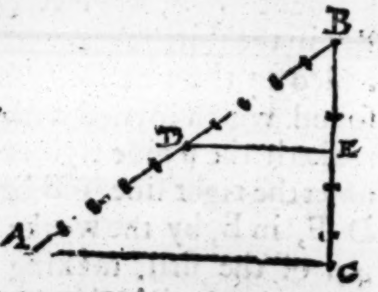
For the Triangles T D E, and L D C, are equiangled by the worke, and by the 38. of the first: Therefore their sides about the equall angles are proportionall, by the 46. of the first. Nor is it any obstacle that the right lines D C, and D E, are diuided into lesser parts, then the right lines D L, and D T, because the Radius D E, is lesser then the Radius G H; with which Radius G H, the right lines D L, and D T, are diuided into equall parts.

For it is no matter into how many parts soeuer one or another side of the plaine Triangle be diuided: So that the like side be divided with the like, into the same equall parts, that is the perpendiculer with the perpendiculer, the hipothenusa with the hipothenusa, and the base with the base.

As for Example; In the Triangles A B C, and D B E. It matters not whether I shall say:

As A B, 10. to D B, 5. So is B C, 3. to B E, $1\frac{1}{2}$. Or,

As *A B*, 5. to D B, $2\frac{1}{2}$. So is B C, 3. to B E, $1\frac{1}{2}$.



Consectarie, *By this declaration and demonstration it appeareth: if the angle giuen at* B. *be a right angle, and his versed sine* E B, *the Radius; in that case there is no need of either multiplication or diuision: but by addition and subtraction onely, the sine of the Complement of the third side may be found out, which briefe rule of calculating of Triangles is more precious then any gold.*

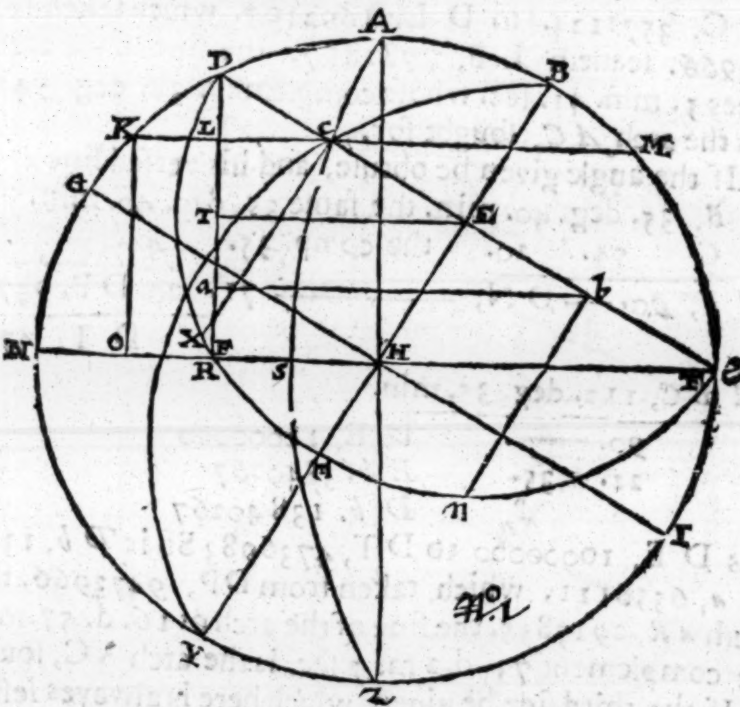
And yet it may be made more briefe, if in the second scheme the sine V F, be not subtracted from the sine D P: *but contrariwise, the right line* D I, *equall to the sine* V F, *bee added to the sine* D P; *For then the halfe of the right line* P I, *shall be presently the sine* T B *sought for.*

And if in the third Scheme, the sine V F, *be not added to the sine* D P, *but on the other side, to wit, the right line* D x, *bee taken from it, then also the halfe of the right line* x P, *shall be now the sine* T P, *sought for.*

The illustration by Numbers.

The first kind of Examples. *Where two sides given both together being equall to a Quadrant; together with the Angle comprehended by them; the third side is sought for. Or contrarily; the third side being also given, the angle opposite thereunto is demanded.*

In the first Scheme No. 1.



1. If the angle given, be a right angle; and his versed sine DE.
A B, 35 deg. 40 min. the same 35 deg. 40 min.
B C, 54 deg. 20 min. the Complem. 35 d. 40 m.

A F, 90 deg. — D N, — 71 d. 20 m. D P, 9473966
D T, or T R, is ———————— 4736983, the sine of the arch of 28 deg. 16 m. 29 sec. whose Complem. 16 deg. 43 min. 31 sec. is the arch A C, sought for.

2 If the angle given, be acute, and his versed sine D C.
A B. 35 deg. 40 min, The same 35. d. 40. m.
B C, 54. deg. 20 min, The Compl. 35. d. 40. m.

A F, 90. deg. — D N. is 71, d. 20 m, D P, 9473966
D T, 4736983
A B C, 50. deg. the Radius, D E, 10000000
The Comple: 40. deg. the sine is, C E, 6427876
D C, 3572124

As E D, — 10000000. to D T, 4736983. So is D C, 3572124. to D L, 1692109. which taken from D P, 9473966. leaueth L P, 7781857. the sine of the arch of 51. degrees 5. min. 41. sec. whose complement 38. deg. 54 min. 19. sec. is the arch *A C*, sought for.

3 If the angle given be obtuse, and his versed sine *D b*.
A B, 35. deg. 40. min. the same 35. deg. 40. min.
B C 54. 20. the comp. 35. 44.
A F, 90. — D N, — 71. 20. D P, 9473966
D T, 4736983

A B C, 112. deg. 35. min.
90. — : D E, 10000000
22. 35. *E b*. 3840267
D b. 13840267

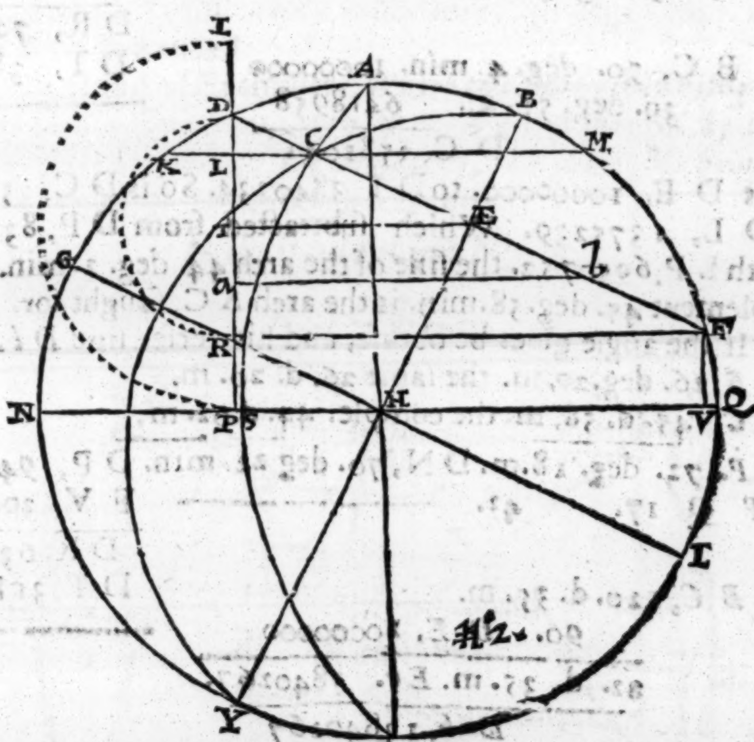
As D E, 10000000 to D T, 4736983 So is *D b*, 13840267 to *D a*, 6556111, which taken from DP, 9473966. there remaineth *a R*, 2917855. the sine of the arch of 16. d. 57. m. 53. se. whose complement 73. d. 2 m. 7 sec. is the arch A C, sought for.

4 If the third side be giuen, which here is alwayes lesser then a quadrant. As for example, If the side *A C*, be giuen, and the angle, *A B C*, be sought for.
A B, 35. d. 40. min. the same 35. d. 40. min.
B C, 54. 20. the comp: 35. 40.
A F, 90. — *D N*, — 71 — 29. *D P*, 9473966
A C, 38. d. 54. m. 19. sec. — *D T*. 4736983
Com. 51. 5. 41. — *L P*, 7781857
D L, 1692109

As D T, 4736983. to D E, 10000000. So is D L, 1692109 to D C, 3572124. which taken from D E. 10000000. leaueth C E. 6427876. the sine of the arch of 40 deg. whose Complement 50. deg. is the angle A B C, sought for.

The second sort of Examples. Where the two sides being giuen both together lesse then a quadrant, with the angle comprehended of them: the third side is required. Or Contrarily, the third side being also giuen, the angle opposite thereunto is demanded.

In the second Scheme N°. 2.



¶ If the angle giuen be a right angle, and his versed sine D E,
A B, 28. deg. 51 min. the same 28. deg. 15. min.
B C, 40. 30. the Compl. 49. 30.

A F 68.

A F, 68. deg. 45. m. D N. 77. deg. 45. m. D P, 9772311
F Q. 21, deg. 15. m, E V. Or P R, D r, 3624280

P r, 13396591

T P, 6698345

Being the sine of the arch of 42. deg. 3. min 15. sec. whose complement 47. deg 56, m. 45, sec. is the arch A C required.

2 If the angle giuen be acute, and his versed sine D C.

A B, 26, deg. 20. m. The same 26. d. 20. m.
B C, 59. 58. The compl. 30. 02.

A F, 86. deg. 18. min. D N: 56. deg. 22. m. D P, 8325991
F Q. 03. deg. 42. min. — — V F, 645323

D R, 7680668

A B C, 50. deg. 4. min. 10000000 D T, 3840334
39. deg. 56. m. 6418958

D C, 3581042

As D E, 10000000. to D T, 3840334. So is D C, 3581042 to D L, 1375239. Which subtracted from D P, 8325991. leaueth L P. 6950752. the sine of the arch 44. deg. 2. min. whose complement 45. deg. 58. min. is the arch A C, sought for.

3 If the angle giues be obtuse, and his versed sine D b.

A B, 26. deg. 20, m. the same 26. d. 20. m.
B C, 45. d. 58, m. the comple. 44. d. 02. m.

A F. 72. deg. 18. m. D N, 70. deg 22. min. D P, 9418621
F Q 17. 42. — F V, 3040331

D R, 6378290

A B C, 120. d. 35. m. D T, 3189145
90. D E, 10000000

32. d. 35. m. E b. 3840267.

D b, 13840267

As D E, 10000000. to D T, 3189145. So is D b. 13840267 to D a 4413861. Which subtracted from D P, 9418621, leaueth a P, 5004760. the sine of the arch of 30. deg. 1. min. 53. sec. Whose complement 59. d. 58. m. 7. sec. is the arch A C, required.

4 If the third side be giuen: which also in these kind of Examples is alwayes lesse then a Quadrant, as for example: If the side A C, be giuen, and the angle A B C, be demanded.

A B,

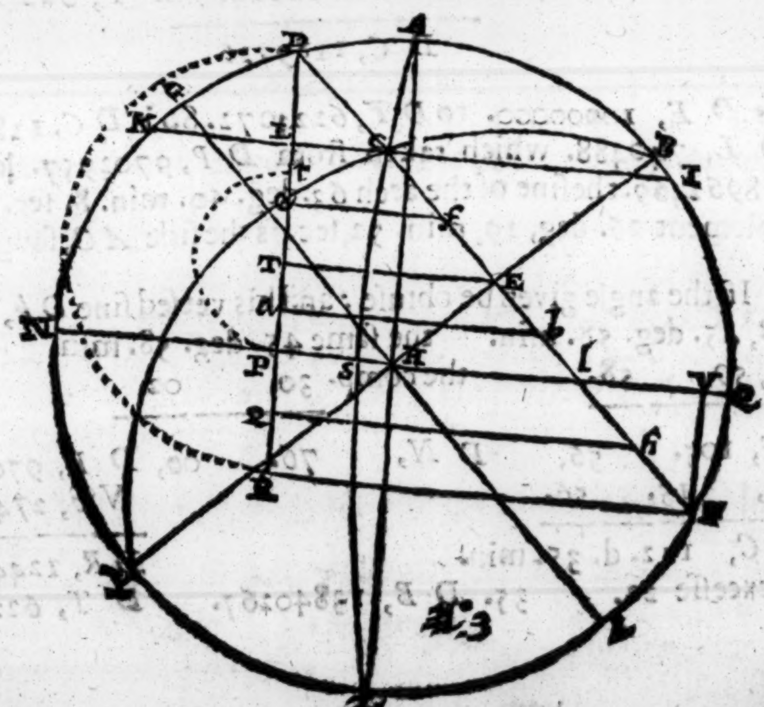
A B, 26 deg. 20 m. The same 26 d. 20 m.
B C, 59. 58. The comp. 30. 02.

A F, 86 deg. 18 min. D N. 56 deg. 22 m. D P, 8325991
F Q, 3. 42. ——————— V F, 645323
D f, 7680668
A C, 45 deg. 58 m. ——————— D T, 3840334
44. 02. ——————— L P, 6950752
D L, 1375239

As D T, 3840334, to D E, 10000000. So is D L, 1375239 to D C, 3581042. the versed sine : which taken from D E, 10000000. leaveth C E, 6418958. the right sine of the angle C B E, 39 deg. 56 min. whose complement 50 deg. 4 min. is the angle A B C, demanded.

The third kind of Examples. *Where two sides given being together more then a Quadrant, with an angle comprehended by them, the third side is demanded : Or contrarily, the third side being also given, the angle opposite thereunto is required.*

In the third Scheme N°. 3.



E 3

1. If the angle given be a right angle, and his versed sine *DE*,
AB, 40 deg. 06 min, The same 40 deg. 06.
BC, 72. 12. The comp. 17. 48.

AF, 112. 18. *DN*, 57. 54. *DP*, 8471219
QF, 22. 18. *VF*, *PR*, or *Dr*, 3794562

rP, 4676657
TP, 2338328

The sine of the arch, of 13. deg. 31. min. 22. sec, whose complement *AC*, 76. deg. 28. min. 38. sec. is the third side, demanded.

2 If the angle giuen be acute, and his versed sine DC.
AB, 45. d. 58. min. The same 45. d. 58. min.
BC, 59. 58 The compl. 30. 02.

A F, 105. 56. DN, 76. — DP, 2702957
QF, 15. 56. VF, 2745187

ABC, 28 d. 14, m. DE, 10000000. DR, 12448144
61. 46. CE, 8810284. *DT*, 6224072

DC, 1189716.

As *DE*, 10000000. to *DT*, 6224072. So is *DC*, 1189716 to *DL*, 740488. which taken from *DP*, 9702957. leaueth LP, 8962469. the sine of the arch 63. deg. 40. min. 8, sec. whose complement 26. deg, 19, min. 52, sec. is the side *AC*, sought for.

3 If the angle given be obtuse: and his versed sine *Db*,
AB, 45. deg. 58. min. the same 45. deg. 58. min.
BC, 59. 58. the comp. 30, 02.

AF, 105. 56, *DN*, 76. 00, *DP*, 9702957
QF, 15. 56. VF, 2745187

ABC, 112. d. 35. min. *DR*, 12448144
the excesse 22. 35. *DB*, 13840267. *DT*, 6224072

As

As D B, 10000000. to D T, 6224072. So is D b, 13840267. to D a, 8614282. which taken from D P, 9702957. leaueth a P, 1088 675. the sine of the arch of 6. degrees 15. minutes, whose complement 83. degrees 45. minutes is the third side A C, required.

4 If the angle giuen be obtuse, and his versed sine D H,
A B, 45, deg. 58. min. The same 45. deg. 58. min.
B C, 59. 58. The comple: 30. 02.

A F, 105. 56. D N, 76. D P, 9702957
Q F, 15. 56. V F, 2745187

D R, 12448144

A B C, 170. T D, 6224072
90.

the excesse 80. D b, 19848077.

As D E, 10000000. to D T, 6224072. So is D b, 19848077 to D Q, 12353586. from whence the sine
D P, ——— ——— 9702957. subtracted.

Leaueth P Q, — 2650529. The sine of the arch, of 15 deg. 22. minutes 14 seconds, which added to the quadrant 90. deg. maketh A C, the third side demanded to bee 105. degrees 22. minutes 14. seconds.

Note; If in this case the fourth number bee found all one with the sine D P, as it should bee found, if the versed sine were D I, it is a signe that the third side is a quadrant, because it hath no sine of the complement, or of the excesse. For if you subtract D P, from D P, there remaineth nothing.

5 If the third side, be giuen lesse then a quadrant, and his versed sine L P.
A B, 45. deg. 58. min. The same 45. deg. 58. min.
B C, 59. 58 The com: 30 02.

A F,

A F, 150. deg, 56 min. D N, 76. deg.	—	D P: 9702957
Q F, 15, 56.	———	V F, 2745187
A C, 26. deg. 20, min.	———	D R, 12448144
		T D, 6224072
63. deg 40. min.	———	L P, 8962469
		D L, 740488

As D T, 6224072. to D E, 10000000. So is D L. 740488 to D C, 1189716: Which subtracted from DE, 10000000. there remaineth. C E, 8810284. the sine of the angle C B E, 61. deg: 46. min. Whose complement 28. degr. 14. min, is the angle A B C, required.

6 If the third side bee giuen more then a Quadrant, and the sine of his excesse P Q.

AB, 45. deg. 58. min: The same 45. d. 58. m.
BC, 59. 58. The compl. 30. 02.

A F, 105. deg. 56. deg. D N, 76, deg.	—	D P, 9702957
Q F, 15. 56.	———	V F, 2745187
		D R, 12448144
		D T, 6224072

A C, — 105. deg. 22, m.
The excesse 15. 22. P Q, 2650629
D P, 9702957
D Q, 12353586

As *D T*, 6224072. to D E, 10000000. So is D *Q*, 12353586 to D h, 19848077. the versed sine of the angle A B C, demanded being 170. deg.

The

The vse of the afore going Axiomes. Or

A direction, whereby is shewed, how by the helpe of these 4. Axiomes, which formerly haue beene explained, any demand, in whatsoeuer sphæricall triangle, may very easily be found out.

First Remember, that some sphæricall triangle, is right angled, and some obliquangled. And that of right angled sphæricall triangles some haue 3. some 2. and others onely one right angle.

Therefore if a right angled sphæricall triangle, haue three right angles, those right angles being giuen; their sides also are giuen: And contrarilie, by the 68. of the first.

If the right angled sphæricall triangle haue two right angles: those two right angles being giuen; the 2. sides also opposite to those 2 right angles, are giuen; to wit, 2. quadrants, by the 68. of the first. But if besides, the third side be also given; or the third angle; either of these being giuen, the other also shall be giuen; for that the third side opposite to the third angle, the sides being 2. quadrants; is nothing else but the measure of that angle, by the 58. of the first.

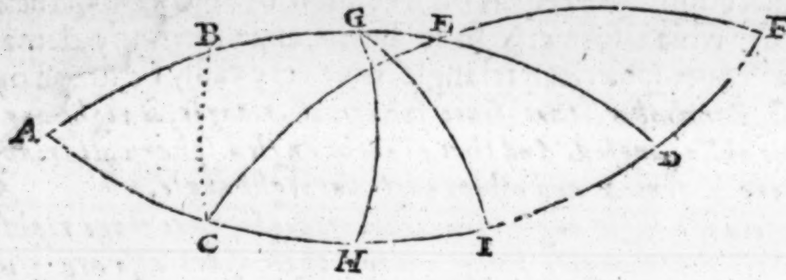
Therefore in these two cases, there is no neede of Trigonometrie. *But if the sphæricall right angled triangle, haue onely one right and the other two oblique angles, in that case* Trigonometrie *is often required.*

And sithence a right angled sphæricall Triangle, of this sort is threefold, for either both the other two angles are acute, or both obtuse or one obtuse and the other acute, by the 63. of the first. My Axiomes shew not the resolution of them, except they haue besides the right angle two acute angles, and by that meanes euerie side lesse then a quadrant by the 65. of the first.

But if a right angled sphæricall Triangle, with two obtuse angles be giuen you to resolue, or with one obtuse and one acute angle, or with two sides either of them more then a quadrant; In stead of that Triangle, you may resolue the lesser Triangle opposite thereunto. As

Let the right angled triangle B D C, right angled at D, and obtuse angled at B and C, be giuen you to resolue; in stead thereof you may resolue the right angled triangle, A B C, opposite from the angle D to the triangle B D C.

For whatsoeuer 3. things are giuen in the triangle B D C, the same 3. things shall be also giuen in the triangle A B C, sithence the angles at A, and D, are equall by the 59. of the first: but the sides



sides A B, and B C, are the complement of the sides B D, and C D, And lastly, the obtuse angle at B, and C, are the Complements of the acute angles, at B, and C, by 60. and 21. of the first.

In like manner, If the triangle C E D, right angled at D, obtuse angled at E, and acute angled at C, bee giuen you to resolue, in stead thereof you may resolue the triangle E D F, opposite to the triangle E C D, from the angle C.

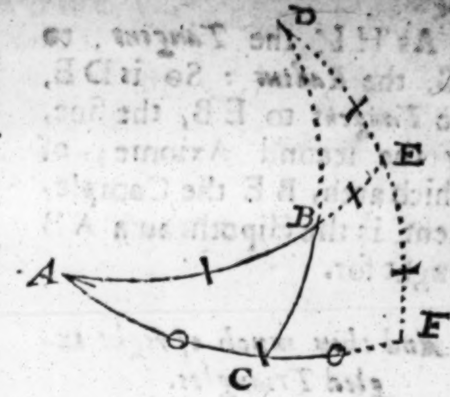
But if a right angled sphericall Triangle, with two acute angles, or with all the three sides, euery one of them beeing lesse then a quadrant, be giuen you to resolue; therein nothing can be demanded that you may not find by the helpe of a few of my Axiomes, out of whatsoeuer three things giuen, either with one multiplication, or diuision, and sometimes also without any multiplication or diuision, by addition and subtraction onely: Prouided alwaies that if in the Triangle propounded, a sufficient proportion for the resolution bee not apparent betwixt the things giuen, and the things demanded, you may then continue euery of the sides untill quadrants. And so conclude the whole figure in a quadrant.

This beeing done, in the Complements of the sides, and angles giuen and required, you shall find some proportion, fitting to your purpose.

As for Example.

In the former of the succeeding figure, in the triangle *A B C*, by the side A B, and the angles B A C, and A C B, the side A C, is required: Because there is no proportion, shewed in these things giuen and demanded, whereof mention is made in the Axiomes of proportions; Therefore you may continue euery of the sides, vnto quadrants, and conclude the whole figure in

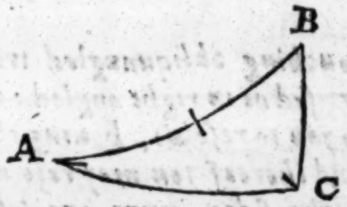
in the quadrant, *D F*, after this manner; which continuation being made in the *B D E*, and C *D F*, such a proportion is giuen, as it is set downe in the second Axiome.

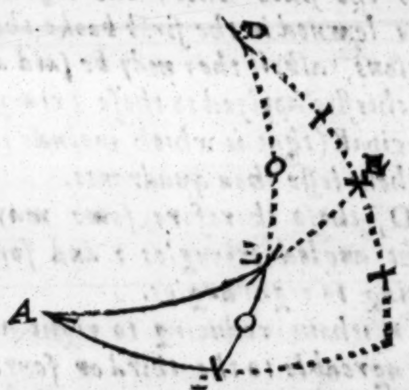


Therefore by that Axiome you shall thus conclude.

As the sine of the base *D E*, to the *Tangent* of the perpendiculer E B, So is the sine of the quadrant *D F*, or the *Radius*, to the *Tangent* of the perpendiculer *FDC*, whose complement is the arch *A* C, required.

Likewise. If in the Triangle A B C, all the angles are given, and the perpendiculer BC, is required; because in these things giuen and sought for, there is no proportion manifest, according to my Axiomes, therefore you may continue the triangle A B C after this manner.



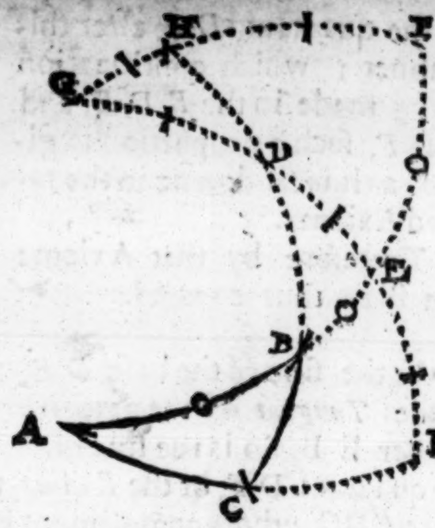


Which being done, in the triangle, D E B, it shall bee. As D B E, to D E. So is D E B, to DB, by the third Axiom; which DB, being knowne; B C, his complement is also knowne.

But if the first continuation be not sufficient, you may also make the second. As you see done in this example; Whereby the three angles giuen to find the hipothenusa, the first continuation is not sufficient; therefore I haue made the second, that is, I haue also continued the triangle B D E, as formerly I had continued the triangle, A B C, which being done, the proportion is.

As H I, the *Tangent*, to I B, the *Radius* : So is D E, the *Tangent* to E B, the sine, by the second Axiome; of which arch, B E the Complement, is the Hipothenusa A B sought for.

And thus much of right angled Triangles.



Of oblique angled Triangles.

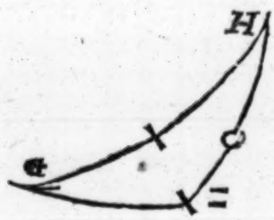
Touching obliquangled triangles, you are for the most part to be aduertised as in right angled : to wit, if any obliquangled triangle be giuen you to resolue, hauing the sides euery one more then quadrants in stead thereof you may resolue the triangle opposite thereunto, that hath the sides euery one lesse than quadrants : which opposite you haue learned in the first booke, the 60, Pro : For my Axiomes of proportions, albeit they may be said after a sort to be generall; yet they are chiefly applyed to those Triangles, whereof euery side, or two of the principall (that is which include the angle giuen or sought) are euery of them lesse then quadrants.

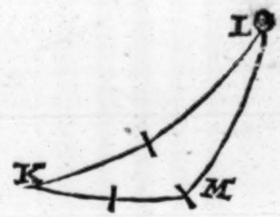
Of these therefore some may bee resolued without reduction to right angled triangles : and some cannot bee resolued without reducing to right angles.

Without reducing to right angles, they may bee resolued which are agreable to the third or fourth Axiomes of proportions.

Those are agreeable to the third Axiome, wherein, by the 2. sides with the angle opposite to one of them, the angle opposite to the other of them is required : Or contrarily by the two angles, with one side opposite to one of them; the other side opposite to the other of them is demanded. As in these Diagrams following. Wherein

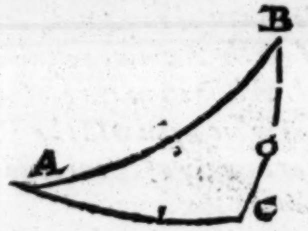
As G I H, *to* G H;
So is H G I, *to* H I.
And as K L, *to* K M L;
So is K M, *to* K L M.

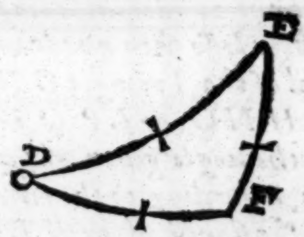


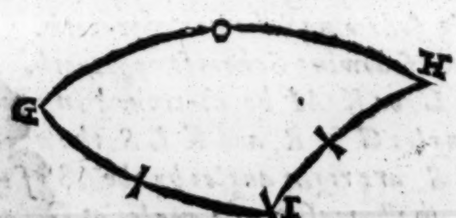


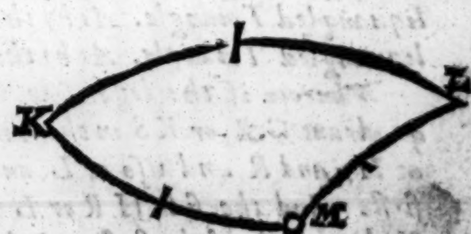
Some are agreeable to the fourth Axiome of Proportions; of themselves; and some accidentally.

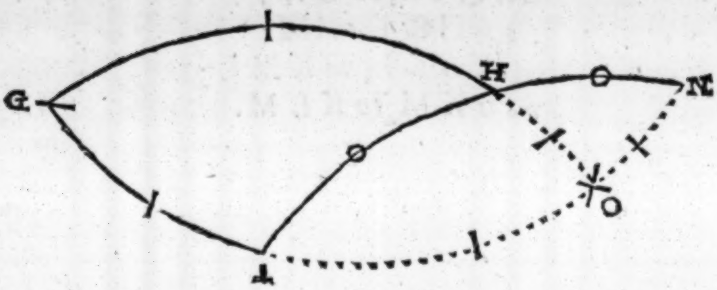
These are of themselves, agreeable to the fourth Axiome of Proportions; Wherein by the two sides given, every of them lesse then Quadrants, together with the angle comprehended by them; eyther the third side is demanded. Or contrarily, by all the three sides given, any angle comprehended of two of the sides, every of them being lesse then Quadrants, is required; As in these.

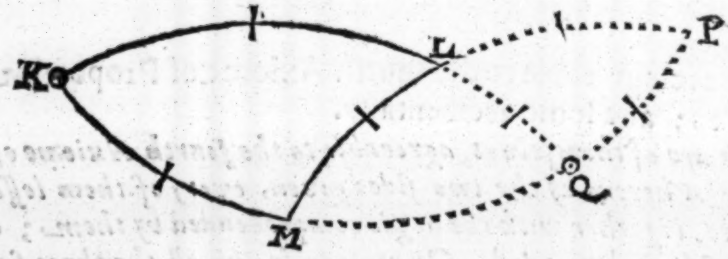












In the 2. latter whereof, which haue the sides, GH, and K L, greater then Quadrants.

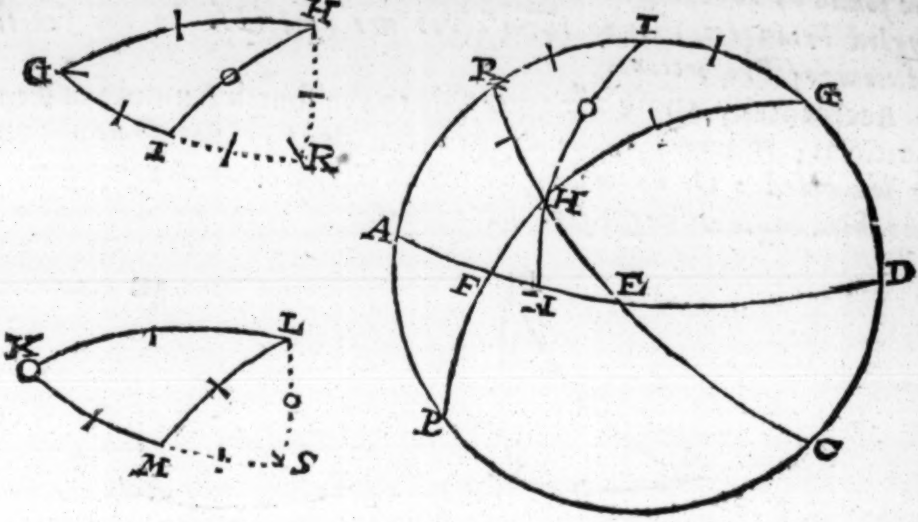
If such a demand be made, as the figures following doe shew: because there, the 2. sides including, the angle giuen or required; are not euery of them lesse then quadrants, as the fourth Axiome requireth.

In stead of the Triangle G H I, or K L M. you may resolue the Triangle, H N O, or L P Q, in whether of which you will the 2 sides including the angle giuen, or sought for, are euery of them lesse then quadrants, according to the rule of the fourth Axiome.

But if the side G H, or K L, be a quadrant, you need not resolue the obliquangled Triangle. G H I, or K L M, but you may resolue the right angled Triangle, which alwayes lyeth next to such an obliquangled Triangle. As by the 3 following Schemes appeareth.
liquangled Triangle. As by the 3. following Schemes appeareth.

Wherein if the lesser side, G L, or K M, be continued untill the quadrant G R, or K S, in the triangles G H R, and K L S, the angles at H, and R, and also at L, and S, are right angles by the 68 of the first. And the side H R or L S, shall measure the angles at G, and K, by the 58. of the first. And so is made the right angled Triangle, I H R, or M L S, of 3 giuen termes; which right angled Triangle

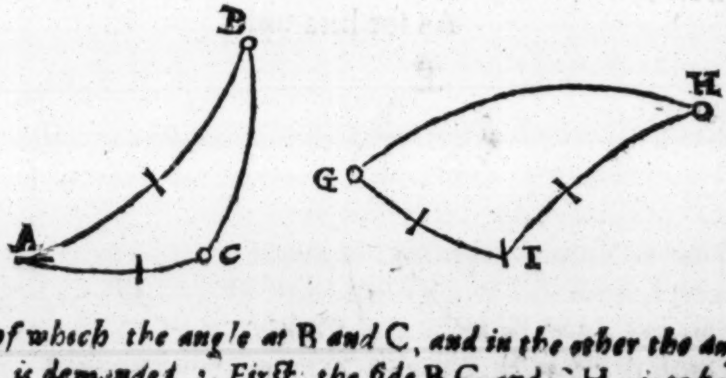
being



being resolved ; the obliquangled Triangle, adjacent thereunto (for that it containeth the Complements of the right angled triangle) shall be resolved.

These things being observed ; the fourth Axiome shall be sufficient : nor shall J need for every case of obliquangled Triangles to make a particuler Axiome, which otherwise should be done.

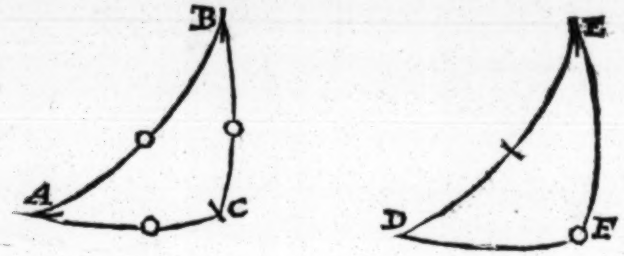
But thus also in this place you are to observe : If the termes given, of an obliquangled Triangle propounded, bee agreeable to the fourth Axiome ; and yet not the termes demanded. As in these.



In one of which the angle at B *and* C, *and in the other the angle at* G *and* H, *is demanded : First, the side* B C, *and* G H *is to be found by the fourth Axiom ; then by that found, any of the other angles may*

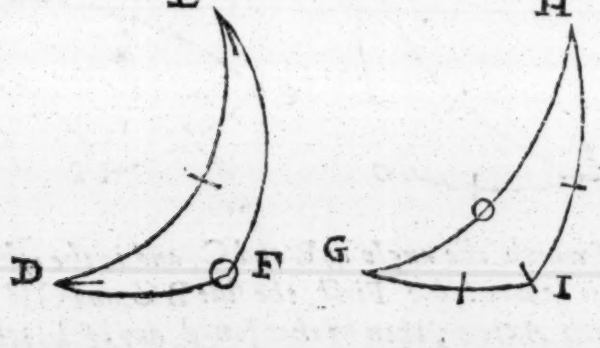
bee found by the third Axiome. And thus much of those Oblique-angled Triangles, that of themselves are agreeable to the fourth Axiome of Proportions.

Accidentally, those are agreeable to the fourth Axiome of Proportions; *wherein eyther by the three Angles given, some one side is demanded : Or by two angles with a side interjacent being given; the third angle is sought for. As in these.*



Which I therefore say accidently, to agree to the fourth Axiome, because otherwise they are not agreeable thereunto, then that the sides may be changed into angles, and the angles into sides; which how the same may be performed, I have shewed in the first Booke, *the* 61 *Prop : Which Proposition, hee that truly understandeth, and well weigheth with himselfe, shall here need no more of that matter. Yet in favour of the Learners which doe not alwayes retaine the speciall points of Rules set downe. I will here insert, and repeat the same : In this changing of Angles and Sides, you must take in stead of the greatest side, and the angle opposite thereunto, the Complements alwayes to a Semicircle; for the reason shewed in the said 61 Prop : of the 1. booke.*

As for Example.



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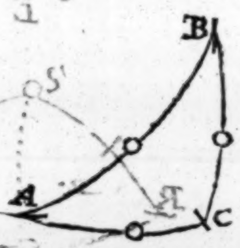
If in the Triangle D E F, you would change the angles into sides; and contrarily, the Triangle will thereupon be such, as is the Triangle G H I. Whereupon it appeareth in the calculation, you are not to take the versed sine of the side D E, but of the complement to a Semi-circle, which Complement answereth to the obtuse angle N I G.

But here also, that hath place which I have fore-warned you of touching oblique angled Triangles, which of themselues are agreable to the fourth Axiome; to wit, if the termes giuen are agreable to the fourth Axiom, but not the terme demanded:

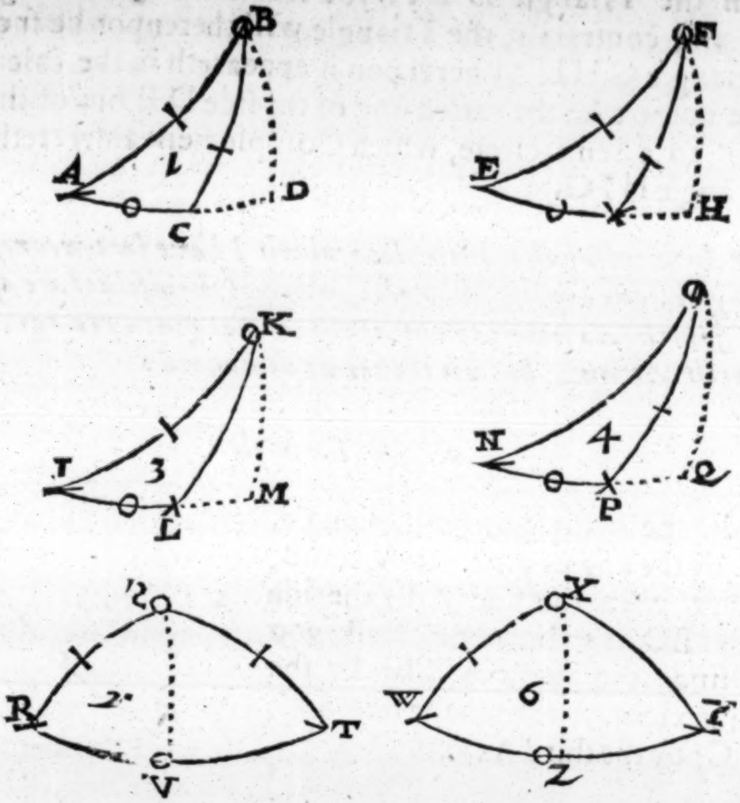
As for Example.

If in the oblique-angled Triangle. A BC, by the angles giuen, at A, and B, together with the side A B, the side AC, and BC, is to be found. First, you must finde the angle A C B, by the fourth Axiom. And then the side A C, or B C, by the third Axiom.



Now there remaineth those obliquangled Triangles, which neither are agreeable to the third nor the fourth Axiomes of Proportions: to wit those, wherein either by the two giuen sides, and an angle opposite to one of them being also giuen: the angle opposite to neither of them, or the side opposite to the unknowne angle is demanded: or contrarily, By the two angles giuen, and a side opposite to one of them being also giuen: the side opposite to neither of them, or the angle opposite to the unknowne side is required.

These cannot be resolued, but by being reduced to right angled Triangles. And they are reduced to right angled Triangles, by letting fall a perpendiculer, which perpendiculer falleth without, or within the Triangle: it falleth without the Triangle, if it be let fall from an acute angle: it falleth within the Triangle, if it be let fall from an obtuse angle: Howsoeuer it falleth, it is alwayes opposite to the knowne angle, and is found by the third Axiom, after this manner.



1 As A D B, to A B, So is D A B to D B.
2 As G H F, to G F, So is H G F, H F.
3 As I M K, to I K, So is M I K, to M K.
4 As P Q O, to P O, So is O P Q, to O Q.
5 As R V S, to R S; So is V R S, to V S.
6 As W Z X, to W X; So is Z W X, to Z X.

And the perpendiculars B D. F H, K M, *&c. in all these oblique-angled Triangles being found, you haue two right angled Triangles of three termes given.*

As for Example.

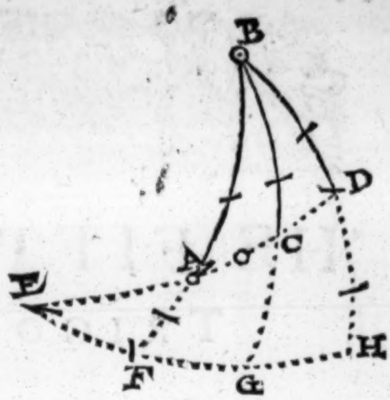
In the first kind *A B D*, and *D C B*,

In the second, *E F H*, and *G F H*, and so forwards.

By helpe of which right angled Triangles, whatsoever is required in the obliquangled Triangles adioyning, is very easily found out, especially if every side be continued to a Quadrant after this manner.

Which

Which continuation being made, If by A B, B C, *and* B A C, *given; I demand* A C, *I say by the first Axiom.*



1 As H D, to D E. So is F A, to A E, which taken from E D, there remaineth A D.

2 As H D, to D E. So is G C, to C E, whose Complement is C D, which taken from A D, there remaineth the arch A C.

But if by the same termes giuen, I would find the angle A B C, *I say by the second Axiom.*

1 As D H, to H E. So is A F, to F E, which taken from E H, there remaineth F H,

2 As D H, to H E, So is C G, to G E, whose Complement is G H, which taken from F H, there remaineth F G, the measure of the angle AB C, demanded. The rest, vse will teach you.

The end of the fourth Booke.

THE FIFTH BOOKE OF TRIGONOMETRIA.

By B. P.

Of the briefe Rules and varieties in the Calculation of Trigonometria.

IN the foure former Bookes I haue set downe the Rules necessary in Trigonometrie, In this fifth and last Booke I will treat of certaine briefe Rules and varieties in the Calculation of Trigonometrie, which although they are not of necessitie, yet they are very pleasant in the vse thereof.

The briefe Rules in the calculation of Trigonometrie are principally, Sixe.

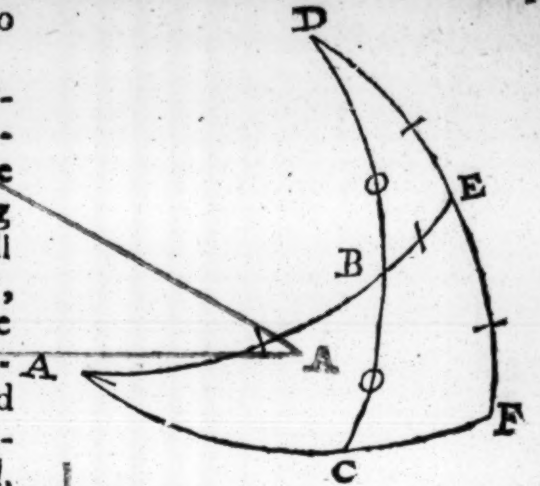
The first briefe Rule.

In the Rule of Proportion, wherein alwayes there ought to be three tearmes giuen: If the first place be Radius, and the second and third a Sine, how to auoyd both multiplication and diuision.

The Rule. *In stead of the two Sines giuen besides the Radius: take the Complements of the arches answering to those sines, and you shall haue a sphæricall Triangle right angled, agreeable to the fourth Axiome of sphæricall Triangles, and so to bee resolued by* Prosthaphæricis.

As for Example.

If such a Proposition bee giuen as Radius A E, to the sine of E F,



EF, so is the sine of A B, to the sine of B C.

In stead of the given arches A B, and E F, in the second and third place, take the Complements of them being B E, and E D; and you shall have the Triangle B E D, right-angled at E. By helpe whereof you shall find (without any multiplication and division by the fourth Axiome) the side B C demanded, being the Complement of the side D B.

Then let the side A B, be 42 deg. —
The side E F, is —— 48 deg. 25 min.
Then the side B E, shall be 48 deg. —
And the side D E, shall be 41 deg. 35 min.

Which things being thus had, I thus proceed:

D E, 41 deg. 35 min. the same 41 d. 35 min.
B E, 48. —— the Compl. 42. —

89. 35. — 83. 35. The sine is 9937354
— 25. —— The sine is - 727721

10010075

The sine of the arch B C, 30. deg. 2. min: — 5005037

The second briefe Rule.

If in the first place be a sine, and the second or third the Radius, to avoid division by bringing the Radius into the first place.

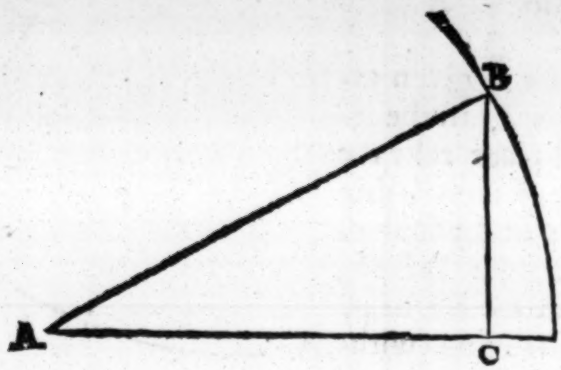
The Rule. *In stead of the Sine put in the first place, take the Secant of the Complement, so shall you have your desire.*

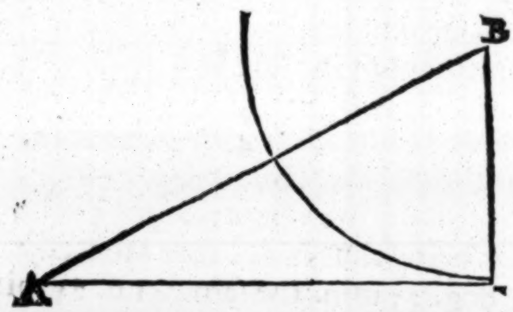
For as the sine is to the Radius: So is the Radius to the Secant of the Compement.

As for Example.

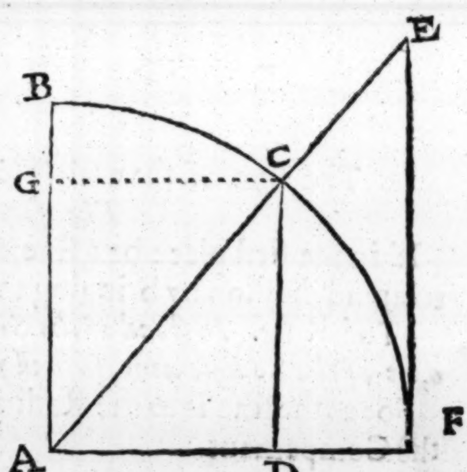
As, B C, the sine of the angle B A C. is to A B, the Radius: So is B C, the Radius to A B, the Secant of the Complement A B C. by the first of plaine Triangles.

The





The same also may be thus demonstrated.



As A D, the sine of the angle A C D, to A C, the Radius: So is A F, Radius to A E, the Secant of the Complement C A D.

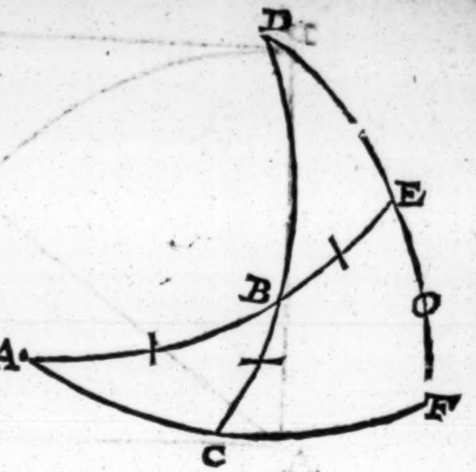
Example.

If this proportion be given: As the sine of the arch A B, 42. deg. is to the sine of the arch B C, 30 deg. 2 min. So is the sine of the arch A E; that is the Radius, to the sine of the arch E F.

In stead of the sine of the arch A B, 42 deg. take the Secant of the Complement 48 deg. and you shall have the proportion thus:

As

As the Radius 10000000. is to the Secant of 48.degrees 14944765. So is the sine of the arch B C, 30.degrees two minutes; to wit, 5005037. to the sine of the arch E F, 7479910. being 48. deg.25. min.



The third briefe Rule.

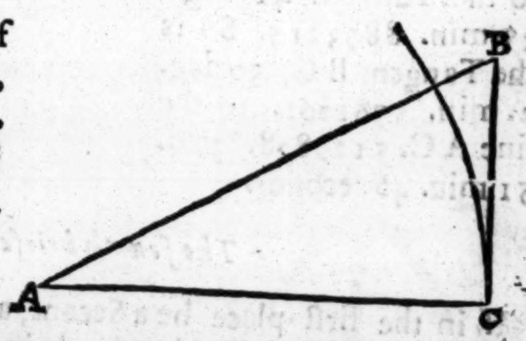
If in the first place bee a Tangent, and in the second, or third a Radius; to auoyd diuision by bringing the Radius into the first place.

The Rule: *In stead of the Tangent put in the first place, take the Tangent of the Complement, and you haue your desire.*

For as the Tangent to the Radius: So is the Radius to the Tangent of the Complement.

As for Example.

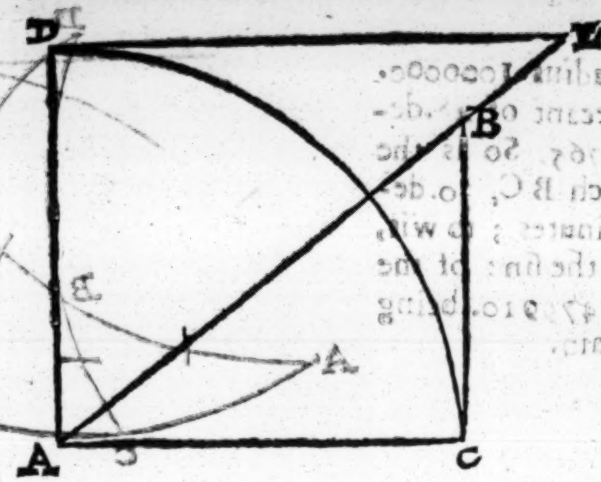
As B C, the Tangent of the angle B A C, is to A C, the Radius: So is B C, the Radius to A C, the Tangent of the Complement A B C, by the first of plaine Triangles.



The same also may be thus demonstrated.

As B C, the Tangent to *A C*, the Radius: So is *A D*, the Radius to *D E*, the Tangent of the Complement.

Example. If this peoportion were giuen, as the Tangent EF, 48. deg.



48 deg. 25 min. to the Radius A F. So is B C, the Tangent 30 deg. 2 min. to the fine A C.

In stead of the Tangent E F, 48 deg. 25 min. take the Tangent of the Complement 41 degr. 35 min. and then your proportion will be such:

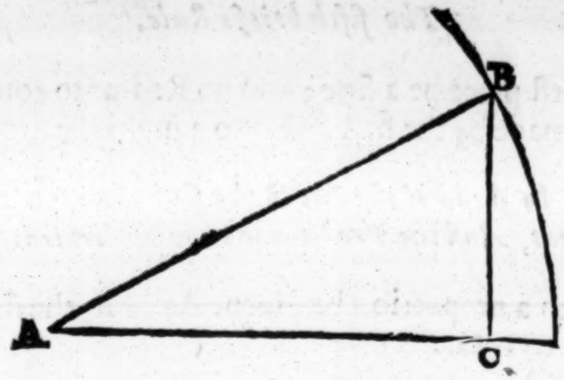
As the Radius 10000000 to the Tangent 41 degr. 35 min. 8873215. So is the Tangent B C, 30 deg. 2. min. 5781262. to the fine A C. 5129838. 30 d. 51 min. 46 seconds.

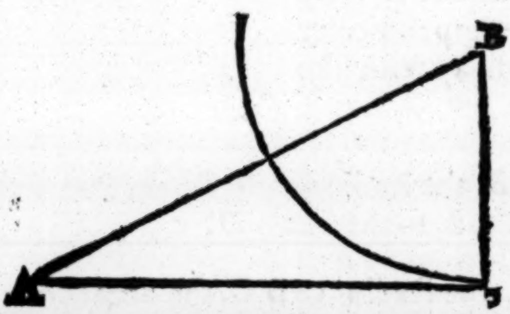
The fourth briefe Rule.

If in the first place be a Secant, and in the second or third the Radius, to auoyd diuision by bringing the Radius into the first place.

The Rule. *In stead of Secant put in the first place & take the fine of the Complement, and you shall haue a proportion wherein the Radius shall bee in the first place.*

For

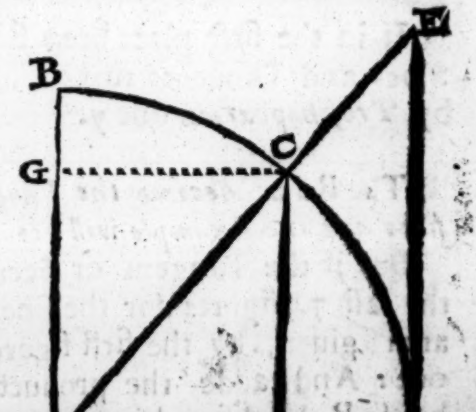




For. As the Secant is to the Radius; So is the Radius to the sine of the Complement.

As for Example.

As A B, the Secant of the angle A B C, is to B C, the Radius; So is A B the Radius, to B C, the sine of the Complement B A C, by the first of plaine Triangles.



The same may bee alse thus Demonstrated.

As the Secant A E, is to the Radius A F. So is the Radius A C, to the sine of the Complement A D.

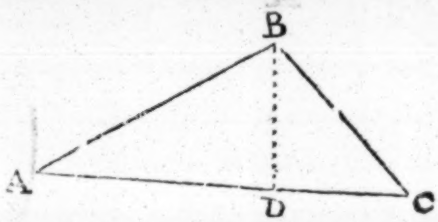
The

The fifth briefe Rule.

If in the first place be a fine; and no Radius to conuert, the diuision to be made by the first fine into a multiplication.

The Rule. *In stead of the fine in the first place, put the Secant of the Complement, And the Problem shall be performed.*

For if such a proportion be giuen. As *A B*, the fine of the angle A C B, is to B C. the fine of the angle B A C. So is the side A B to the side B C: By letting fall the perpendiculer B D. you shall say with like effect.



1 As *A B*, the Radius, to *B D*, the fine of the angle B A C. So is the side A B, to the side B D.

2 As B D. the Radius to B C, the Secant of the angle D B C. being the Complement of the angle A C B, or *D C B*. So is the side *B D*, to the side B C, by the first of plaine Triangles.

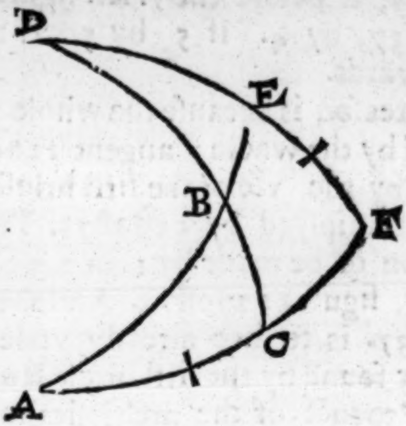
The sixth briefe Rule.

If in the first place bee a Radius; and in the second and third Sines and Tangents mixed, Secants; to resolue the Problem by *Prosthaphæresis* onely.

The Rule. *Accompt the Tangents and Secants in the place of the fines and the Example will bee agreeable to the first brief Rule.*

But if the Tangent or Secant haue more then 7 figures, take the last 7. figures for the fine, and multiply the fine of the other arch giuen, by the first figure, or figures: if there be more then one: And adde the product to the number found by the first briefe Rule. After this manner.

If



If ſuch a proportion were giuen; as the Radius A F, to the Tangent, E F, of the angle E A F, 48.d.25.m. which Tangent is 11269872. So is the ſine of the arch *A C*: (which is the arch of 30.d. 51. m. 46. ſec.) 5129837. to the Tangent of the arch B C. Take for ẏ ſine 1269872 the ſeauen laſt figures of the Tangent 11269872; and take out of the Table the arch anſwering to that ſine, being 7. degr. 17.min.44.ſec. Then proceed thus by the firſt briefe Rule.

Of the arch 7.d. 17 m. 44 ſec. the Compl: is 82 d. 42 m. 16.ſec.
Of the arch 30. 51. 46. the Compl: is 59. 8. 14.

Therefore according to the fourth Axiome of ſphæricall Triangles.

The leſſer ſide is 59.d.8.m.14.ſec. the ſame is 59.d.08,m.12.ſec.
the greater ſide is 82. 42. 16. the Compl. is 07 17. 44.
the ſum is 141 d. 50 m. 30. ſ. 66 d. 25.m. 58 ſ. the ſine 9165916
The exceſſe 51 d.50 m.30 ſec. the ſine whereof is 7863064

1302852

The number found by the firſt briefe rule is 651426
To which adde the ſine of the other arch giuen, 5129837
The totall is the Tangent of the arch of 30. deg. 2. min. 5781263 for the arch B C. required.

Note. If the giuen Tangent were 21269872, after the order of the firſt briefe Rule by the laſt ſeauen figures, it is 1269872 & you ſhould multiply the ſine of the other arch giuen 3129837 by two and the product you ſhould adde to 651426. the number found by the firſt briefe Rule.

But if the Tangent were ſuch 31269872: after the practice of the firſt briefe Rule by the laſt 78. figures 1269172. you ſhould multiply the ſine of the other giuen arch 5129837. by three.

S Laſtly,

Lastly, if before the 7. last figures were 4, you should multiply 5129837, by 4. if 5, by 5: If 12, by 12. If 213. by 213. and so forwards.

The reason is because the whole sine 5129837. was to be multiplyed by the whole Tangent 11269872.

But by the vse of the first briefe Rule, the sine 5129837, was onely multiplyed by 1269872. Then there remained the multiplication to be made by 1, or 2, or 3. or whatsoeuer went before those 7, figures 1269872. And therefore the product by 10, and 5129837. is to bee directly vnder-written vnder 651426. the number found by the first briefe Rule; because that found number is the Product of the multiplication of the sine 5129837. and 1269872. diuided by the Radius; which Product if it were not diuided by the Radius should stand thus, 6514260000000.

Then because the last multiplyer 1. is in the eight place toward the left hand therfore also the product of the multiplier 5129837 shall necessarily be so vnder written, that his last number be in the eight place, the last but one in the ninth place; and so forwards after this manner. 6514260000000.

5129837.

The seauenth briefe Rule.

Whatsoeuer tearmes are giuen; to find out the demand by *Prosthapharisis* onely.

The Rule. *That you may alwayes haue the Radius in the first place; Worke by the second, third, fourth, or fift briefe Rule, then performe the rest by the first or sixth briefe Rule.*

Of the varieties in generall of Trigonometricall calculation.

In the resolution of Triangles, especially of sphaericall; one and the same demand oftentimes by the same things giuen may bee found out sundry and diuers wayes: Whereof there are foure reasons, euery of which I will vnfold in seuerall Theorems.

The first Theorem of the variety of Trigonometricall calculation.

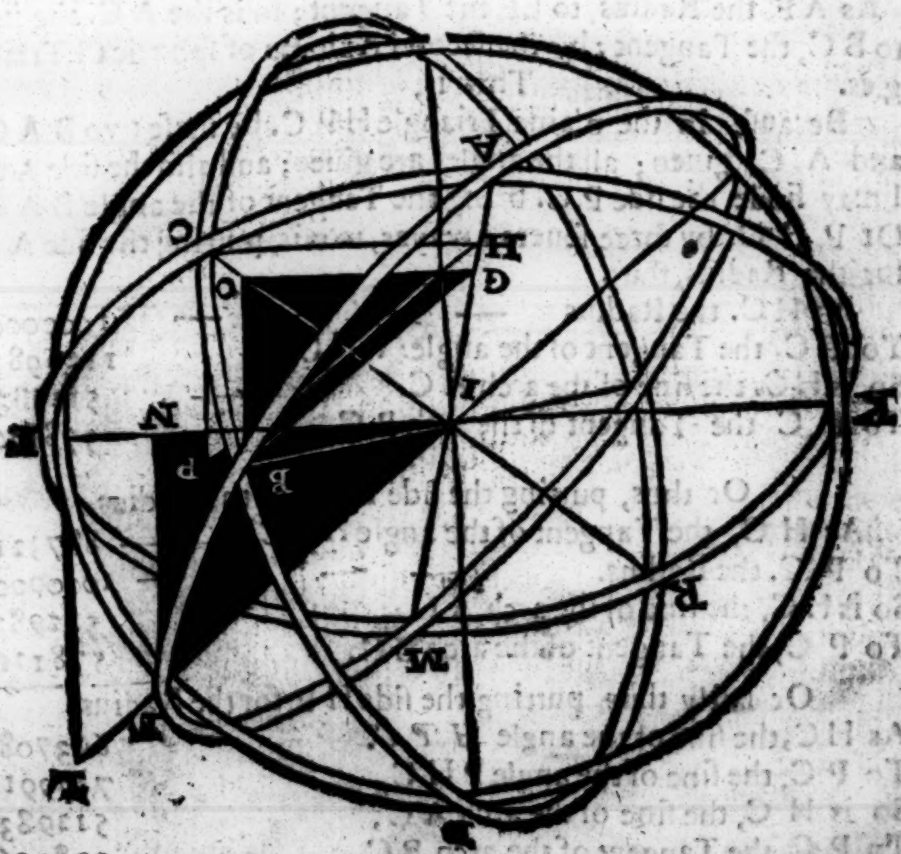
Euery proportion of the Radius to the sine, Tangent, or Secant, and contrarily; may be varied three wayes by the first Axiome of plaine Triangles.

Therefore

Therefore in the right angled Triangle A B C, if by the arches B A C, 48 deg. 25 min. and A B, 42 deg. given; the arch B C, be demanded. Because this Proportion is given, As A E, the Radius, is to E F, the sine; so is A B the sine, to B C the sine, by the first Axiomes of Sphæricall Triangles. That is,

Because in the plaine Triangle G B O, by those two given, B A C and A B; all the angles, and moreover the side G B, to wit; the sine of the arch A B, are given; I may find the side B O, being the sine of the arch B C, three wayes; To say, either thus,

As G B, the Radius,	10000000
To B O, the sine of the angle B A C; or B G O,	7479912
So is G B, the sine of the arch A B,	6691306
To B O, the sine of the arch B C,	5005038



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Or

Or thus.

As G B, the Secant of the angle B G O.	15066852
To B O. the Tangent of the same angle.	11269872
So is G B. the sine of the arch A B,	6691306
To B O. the sine of the arch B C,	5005038

Or lastly thus.

As G B. the Secant of the angle G B O.	13369141
To B O, the Radius	10000000
So is G B. the sine of the arch A B.	6691306
To B O. the sine of the arch B C.	5005038

So in the same sphæricall Triangle A B C. If by B A C, 48, deg. 25, m. and AC, 30. deg, 51. m. 40. sec, giuen : the arch BC. be demanded. Because the proportion is giuen.

As A F. the Radius, to EF the Tangent; so is the A C. the sine to B C, the Tangent : by the second Axiome of sphæricall Triangles. That is,

Because in the plaine Triangle H P C. by these two B A C: and A C, giuen; all the angles are giuen; and also the side A C, I may finde the side P C. being the Tangent of the angle B A C, Or P H C, by three seuerall wayes, to wit, putting the side A C for the Radius, thus.

As H C. the Radius	10000000
To P C. the Tangent of the angle: P H C.	11269872
So is H C. the sine of the arch A C.	5129838
To P C the Tangent of the arch B C.	5781262

Or thus, putting the side P C, for the Radius.

As H C, the Tangent of the angle H P C,	8873215
To P C. the Radius.	10000000
So is H C, the sine of the arch A C,	5129838
To P C, the Tangent of the arch B C.	5781262

Or lastly thus, putting the side H P, for the Radius,

As H C, the sine of the angle *H P C.*	6637087
To P C, the sine of the angle P H C,	7479912
So is H C, the sine of the arch A C,	5129838
To P C, the Tangent of the arch B C,	5781262

The

The second Theorem of the variety of Trigonometricall calculation.

In the rule of Proportion, wherein there are alwayes foure tearmes; three given, the fourth demanded: It is all one whether of the two middle tearmes I shall put in the second or third place.

For it is all one, whether I shall say,

As 2. to 4. so 5. to 10. Or,
As 2. to 5. so 4. to 10.

From hence every Example of the first Theorem may againe be varied three wayes.

The first Example of the first Theorem, was thus:

As the Radius,	10000000
To the sine of the angle *B A C*,	7479912
So is the sine of the arch A B,	6691306
To the sine of the arch *B C*,	5005028

In stead thereof I may now say, using againe the variety of the first Theorem,

As the Radius,	10000000
To the sine of the arch *A B*,	6691306
So is the sine of the angle *B A C*,	7479912
To the sine of the arch *B C*,	5005038

Or, as the Secant of the arch *A B*,	13456327
To the Tangant of the same arch,	9004040
So is the sine of the angle *B A C*,	7479912
To the sine of the arch *B C*,	5005038

Or lastly, as the Secant of the compl: of the arch *A B*,	14944756
To the Radius,	10000000
So is the sine of the angle B *A C*,	7479912
To the sine of the arch B C,	5005038

The second Example of the first Theorem, was thus:

As the Radius,	10000000
To the Tangent of the angle B *A C*,	11269872
So is the sine of the arch *A C*,	5129838
To the Tangent of the arch B C,	5781262

In stead thereof, I will now say: using the variety also of the first Theorem.

As the Radius,	10000000
To the sine of the arch *A C*,	5129838

So is the Tangent of the angle *B A C*,	11266872
To the Tangent of the arch *B C*.	5781262
Or, As the Secant of the arch, *AC*.	11649603
To the Tangent of the same arch	5976055
So is the Tangent of the angle *B AC*,	11269872
To the Tangent of the arch *B C*.	5781262
Or lastly, as the Secant of the Compl: of ẏ arch A C.	19493797
To the Radius,	10000000
So is the Tangent of the angle B A C,	11269838
To the Tangent of the arch B C.	5781262

The third Theorem of the varietie of Trigonometricall calculation.

The sines of the arches and the Secant of the Complements are reciprocally proportionall.

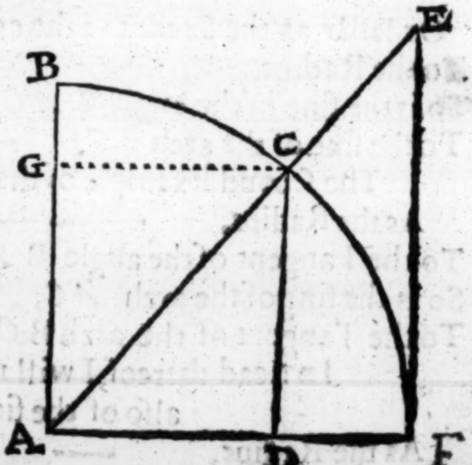
That is, As the sine of the greater arch is to the sine of the lesser arch: So is the Secant of the Complement of the lesser arch to the Secant of the complement of the greater arch.

And in like manner, As the sine of the lesser arch is to the sine of the greater arch: So is the Secant of the Complement of the greater arch, to the Secant of the Complement of the lesser arch.

The reason of this reciprocall proportion, is

Because the Radius is a meane proportionall betweene the sine of any arch of the Secant of the Complement of that arch. That is,

As the sine is to the Radius. So is the Radius to the Secant of the Complement.



As for Example.

As A D, the sine of the arch B C, is to the Radius A C. So is the Radius A F. to A E, the Secant of the Complement C F, by the fourth of the sixt of *Euclide*; Or by the 46 of the first hereof.

Therefore whatsoever sine is multiplyed by the Secant of the Complement, it maketh the square of the Radius.

And thereupon, the plaine figures made of the sines of the arches, and the Secants of the Complements, are all equall to one another: viz. they are equall to one, and the same square of the Radius.

But equall plaine figures haue their sides reciprocally proportionall by the 42. of the first, Therefore as the sine of a greater arch is to the sine of any lesser arch: So is the Secant of the Complement of the lesser arch to the Secant of the Complement of the greater arch.

All this is very easie to be discerned in small numbers: For let the two sines be 4, and 2. the Radius 10.

First, it is manifest, that the Secants of their Complement are 25. and 50. For,

As 4. to 10. so is 10. to 25. And

As 2. to 10. so is 10: to 50.

Then it is manifest, that the Secant of the Complement 2. is to the Secant of the Complement 4; As 4. to 2. For the Secant of the Complement of 2: is 50. And the Secant of the Complement of 4. is 25. Then as 4, is to 2. So is 50, to 25.

In greater numbers is the same reason. For let the two sines giuen, be 6691306. and 5005038. and let the Secants of the Complements be demanded after this manner.

As 6691306. to 10000000. so is 10000000. to 14944765: and

As 5005038. to 10000000. so is 10000000. to 19979868.

It is manifest after these numbers found, that.

As 6691306. is to 5005038. so is 19799868. to 14944765.

Hereupon, I may vary againe the first example of the first Theorem, sixe wayes.

For if by the first Theorem, I shall take this proportion:

As the Radius, 10000000. to the sine of the angle B A C. 7479912. inverting that proportion, I may say, vsing also the variety of the first Theorem: either,

As the sine of the angle B A C:	7479912
To the Radius,	10000000
So is the Secant of the Complement of the arch A B.	14944765
To the Secant of the Complement of the arch B C,	19979868

Or, As the Tangent of the angle B A C. 11269872

 To

To the Secant of the ſame angle, ——— ——— 15066852
So is the Secant of the complement of the arch A B, 14944765
To the Secant of the complement of the arch B C, 19979808

Or laſtly, as the Radius ——— ——— ——— 10000000
To the Secant of the complement of the angle B AC, 13369141
So is the Secant of the complement of the arch, A B. 14944765
To the Secant of the complement of the arch *B C*, 19979868

But if by the Second Theorem, I ſhall take this proportion,
As the Radius 10000000 to the ſine of the arch A B, 6691306
Inverting that proportion, I may ſay by this third Theorem, uſing the variety of the firſt Theorem in like manner; either,

As the ſine of the arch A B, ——— ——— ——— 5691306
To the Radius, ——— ——— ——— 10000000
So is the Secant of the comple: of the angle B A C, 13369141
To the Secant of the complement of the arch B C, 19979868

Or, As the Tangent of the arch A B. ——— ——— 9004040
To the Secant of the ſame arch ——— ——— ——— 13456327
So is the Secant of the comple: of the angle B A C, 13369241
To the Secant of the complement of the arch B C. 19979868

Or, Laſtly, as the Radius, ——— ——— ——— ——— 10000000
To the Secant of the complement ——— ——— AB, 14944765
So is the Secant of the comple: of the angle B A C, 13369141
To the Secant of the complement of the arch B C. 19979868

So by the ſame *B A C*, and *A B*. given. I ſhall find the arch *B C*, 12. wayes; thrice by the firſt Theorem, and againe thrice by the ſecond; and laſtly, ſixe times by the third Theorem.

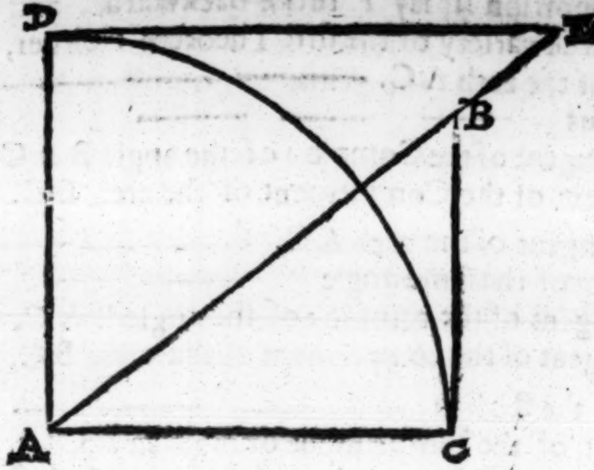
The fourth Theorem of the variety of Trigonometricall calculatton.

The Tangents of Arches, and the Tangents of the Complements are reciprocally proportionall.

That is, as the Tangent of the greater arch, is to the Tangent of the leſſer arch: So is the Tangent of the Complement of the leſſer arch, to the Tangent of the Complement of the greater arch.

And contrarily.

The reaſon of this reciprocall proportion is, the ſame that is in



in the Secants: that is because the Radius is a meane proportion betwixt the Tangent of an arch, and the Tangent of the Complement. For,

As E D to A D. So is *A C* to B C. by the 4. of the sixt of *Euclide*. or by the 46. of the first hereof.

As for example: When the proportion is.

As 11269872. The Tangent of 48. deg. 25. min. to the Radius 10000000: So is the Radius 10000000. to 8873215. the Tangent of the Complement. And,

As 5781262. the Tangent of 30. deg. 2. min. to the Radius 10000000. so is the Radius 10000000. to 17297260. the Tangent of the Complement. It shall be also.

As 11269872. to 5781262. so is. 17297260. to 8873215. Or contrariwise.

As 5781262. to 11269872. so is 8873215, to 17297260.

Hereupon, if by 11269872. the Tangent given, the Tangent 5781262. bee demanded: leaving those Tangents, I may suppose 8873215. the Tangent of the Complement to bee given, and the Tangent of the other Complement 17297260. to be demanded. In taking of which supposition, I invert the proportion of the second example of the second Theorem; Which was thus: As the Radius 10000000. to the sine of the arch *A C*. so is the Tangent of the angle B A C. to the Tangent of the arch B C.

This proportion I say I turne backward, and say, using therewithall the variety of the first Theorem: either,

As the sine of the arch A C.	5129838
To the Radius	10000000
So is the Tangent of the Comple: of the angle B A C.	8873215
To the tangent of the Complement of the arch B C.	17297260
Or, as the tangent of the arch A C.	5976055
To the Secant of the same angle	11649603
So is the tangent of the comple: of the angle B A C.	8873215
To the Tangent of the complement of the arch B C,	17297260
Or lastly, As the Radius	10000000
To the secant of the complement of the arch A C,	19493797
So is the tangent of the comple: of the angle B A C	8873215
To the tangent of the complement of the arch B C	17297260

And so by the same B A C, and A C, given, I shall finde the arch B C, nine wayes, thrice by the first Theorem; thrice by the second, and againe thrice by the fourth Theorem.

Touching the variety of Trigonometricall calculation in particular: concerning the three former Axiomes of plaine Triangles.

The three former Axiomes of plaine Triangles, may happily be more rightly drawne into one, and may thus bee propounded.

The sides are directly proportionall, to the subtenses of the opposite angles.

That is, as the greatest side is in Proportion to the least side: So is the subtense of the greatest angle in proportion to the subtense of the least angle: And so of the rest.

The reason is, because a circle may bee circumscribed to every plaine Triangle: which if it bee done, the sides themselves of the plaine Triangle, are the subtenses of the angles opposite thereunto, as is shewed in the third *Axiome* 3. Booke.

A generall Consectary.

Therefore the subtenses being given, of whatsoever two angles, with a side opposite to one of the angles given: the side also opposite to the other of the given angles is given: And contrarily.

The

The two sides whatsoever being given, with the subtense of any angle opposite to one of these sides given: the subtense also of the angle opposite to the other side, is also given: and by the subtense, the angle it selfe.

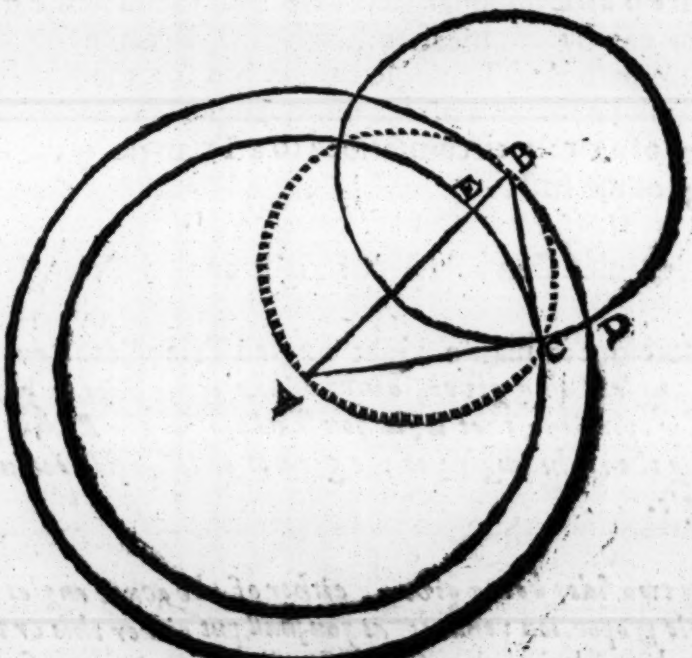
And the subtenses of the angles given, in plaine Triangles, are given three wayes; to wit, either thus.

1. That the side subtending the right angle bee Radius, and the sides including the right angle, sines: Or thus,

2. That the greater side including the right angle be Radius: and the other two sides the tangent and secant of the lesser acute angle. Or lastly thus,

3. That the lesser side concluding the right angle be Radius: and the other two sides; the tangent and secant of the greater acute angle.

As in the plaine right angled Triangle ABC, wherein the sides AB, BC, and AC, are the subtenses of the angles opposite unto them, in respect of the pricked circle ABC. If you put the side AB, for the Radius; the sides BC, and AC, shall be the sines of the angles BAC, and ABC in respect of the Circle BD.



If you put the side *A C*: for the Radius, the side *B C.* shall be the Tangent of the acute angle B A C. and the side B A, shall be the Secant of the same angle in respect of the circle E C.

If you put the side B C for the Radius, the side A C. shall be the Tangeet of the angle acute A B C, and the side *A B.* shall be the Secant of the same angle, in respect of the circle C D.

But in plaine obliquangled Triangles, the angles being given, the subtenses are given by one way, to wit, by the sine onely: For the Tangents and Secants in plaine obliquangled Triangles are of no use, by their definitions.

But the sines are of use in all, because they are the halfe of the subtenses inscribed in a circle: which subtenses of every plaine Triangle may bee made the sides; by the demonstration a foregoing.

But in a plaine right angled Triangle the subtense of every angle cannot be given; For every side of a plaine right angled Triangle, may be put for the Radius, that is 10000000. and so may be accompted for the subtense of the angle opposite, not yet knowne.

But in a plaine obliquangled Triangle, the subtense of a angle not given can by no meanes be given: Because no side of a plaine obliquangled Triangle can be put for the Radius; and that because no side of a plaine obliquangled Triangle can be the Diameter of a circle circumscribed to a Triangle by the first *Con:* of the 53. of the first.

Particuler Consectaries of right angled Triangles.

1. Therefore in plaine right angled Triangles: *one side besides the angles being given, every of the other sides is given by a threefold proportion; that is, as you shall put for the Radius, the side subtending the right angle; or the greater or lesser side including the right angle.*

2. *Any two sides being given, either of the acute angles is given by a double proportion: that is, as you shall put either this or that side for the Radius*, being the subtense of the angle opposite either knowne or unknowne.

Parti-

Particuler Consectaries of obliquangled Triangles:

1 In plaine obliquangled Triangles, *one side being given besides the angles: everie of the other sides is given but by one proportion onely, &c,*

2 Any two sides being given, not simply. b *ut onely two sides being given with an angle opposite to one of them: the angle opposite to the other of them is given.*

This abstract in my opinion was more methodicall, but that reason, which I have laid downe in my third booke for the understanding of young learners, was more fit, in the opinion of those my Schollers, who had some interest in me.

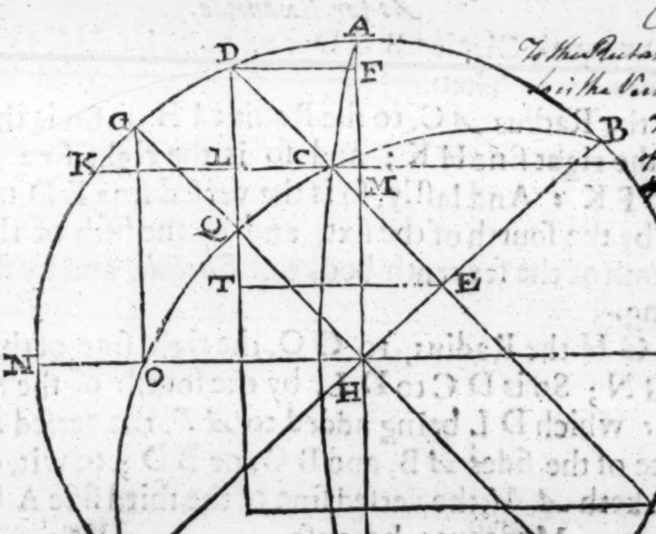
Againe: Of the varietie of Trigonometricall calculation in particuler.

About the fourth Axiome of Sphæricall Triangles.

Regiomontanus, and *Finskius*, and *Landsbergius* following him, doe thus propound the fourth axiome of Sphæricall Triangles.

The square of the Radius, is to the plaine figure made of the right sines of the vnequall sides; As the versed sine of the angle comprehended of the said two sides, is to the difference of the versed sines of the third side, and the difference of the other two sides.

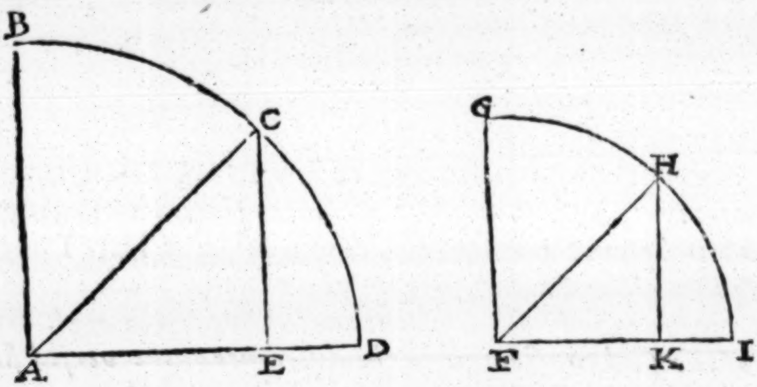
As the square of Radius
To the Rectangle of the Sines of the sides
so is the Versed Sine of the contained Angle
To the difference of the Versed Sines
of the diff.nce of the two sides &
~~of the~~ the third side required



Theis

Their Demonstration, is briefly thus.

1. As G H the Radius, to D E, the right sine of the greater side B C, or B D. So is G Q the versed sine of the angle A B C, in the diameter of a great circle to D C, the same versed sine in the diameter of a circle, to the Paralell: Because in unequall Circles: *As the Radius of one Circle to the Radius of another Circle: So is the Sines aswell right as versed of the one Circle, to sines of like arches, aswell right as versed of the other Circle.*



As for Example.

In the unequall Circles B C D, and G H I: If C D, and H I be like arches. Then,

1. As the Radius *A* C, to the Radius I H: So is the right sine C E, to the right sine H K; and so is the right sine *A E*, to the right sine F K: And lastly, so is the versed sine E D, to the versed sine K I, by the fourth of the sixt, and by the fifth of the fift, and the eleventh of the seaventh booke of *Euclide*, and by the Schemes adjoyning.

2. As G H the Radius, to G O, the right sine of the lesser side A B or G N; So is D C to D L, by the fourth of the 6. booke of *Euclide*: which D L being added to *A F*, the versed sine of the difference of the sides *A* B, and B C, or B D; to wit, of the arch *A D*, maketh *A M*, the versed sine of the third side A C, or *A* K.

Moreover, because

As G H, to D E. So is G Q, to D C. *And*

As G H, to G O; so is D C to D L. It is also by the multiplication of Proportions.

As the plaine figure G H G H, to the plaine D E G O. So is the plaine G Q D C, to the plaine D C D L, And the last two plaines G Q D C, and D C D L. being deuided by their common side D C.

As the plaine G H G H. to the plaine D E G O. So is the side G Q. to the side D L.

Or the first two plaines also being deuided by some common diuisor; to wit, the Radius.

As G H. the Radius to the plaine D E G O. diuided by the Radius: so is the side G Q. to the side D L.

For if: As 10. to 8, so 5. to 4. And,

As 10. to 5. so 4. to 2. Then it shall be,

As 100. to 40. so. 20. to 8. And the last two plaines deuided by their common side 4.

As 100. to 40. so 5. to 2. Or the first 2. plaines being diuided by some common diuisor: viz, by 10.

As 10. to 4. so is 5. to 2.

This the demonstration of *Regiemontanus*, *Finckius*, and *Landsbergius*, altogether certaine and infallible. Which euery man sees that is a Geometrician.

An Example, repeated out of the third kind of my Examples.

	B C. 59, deg. 58 m: the right sine is		8657344
	A B. 45. 58. the right sine is		7189355
Difference	14.		10000000
	76.		9702957
The versed sine of the difference			00229704
	A B C. 28 deg. 14, m.		10000003
	61. 46.		8810284
The versed sine of the angle			1189716

The plaine made of the right sines A B and B C. 62240719312o
The same plaine devided by the Radius, is ——— 6224072

Truely agreeing with the halfe of the right line found of me, by *Prosthaphæricis*.

The Proportion.

As the Radius ——— 10000000: D B
To the plaine of ỹ right sines diuided by ỹ Radius. 6224072. D T,
So is the versed sine of the angle A B C. ——— 1189716. D C,
To the difference ——— 740488. D L.

Which

Which if you adde to the versed sine of the difference of the side,	297043.	A F
Make the versed sine of the third side	1035531,	A M
Which taken from the Radius	10000000.	A H

Leaveth the sine of the Compl: of the third side, 8962469, M H. To which sine the arch K N, 83. degr. 40. m. 8. sec. answereth: the Complement whereof is A K or A C, 26. d 20, m. 52. sec. the arch demanded.

Justus Bergius *in the working of the fourth Axiome, never useth the versed Sines, but alwayes the right sines.*

And first, The angle at B. *being a right angle, hee seeketh out what should be the sine of the Complement of the third side: then if the angle* B. *be acute, hee findeth the difference of that sine from the sine of the oblique-angle, that is the right line* C O, *or* E O, *or* L T, *by such proportion.*

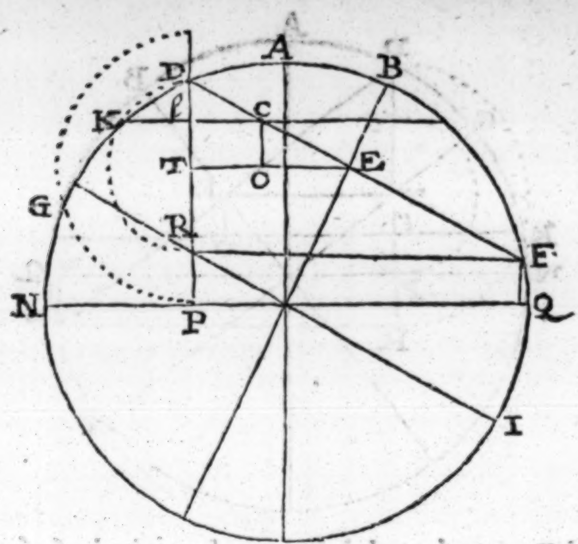
As D E, the Radius to *D T.* the halfe of the right line; So is C E, the sine of the Complement of the angle A B C. to C O.

If the angle at B, *be obtuse; by such proportion:* As *D E*, the Radius, to *D T.* the halfe of the right line, so is E C. the sine of the excesse of the angle *A B C*, to E O.

Which proportion notwithstanding; he resolveth the same without any multiplication and division, by the helpe of my first briefe Rule.

Moreover, the angle A B C. being acute, he alwayes addeth L T. the number found to T P. to make L P. the sine of the Complement of the third side. But the angle ABC *being obtuse, either he subtracteth the found number T L, from T P. that there may remaine L P. the sine of the Complement of the third side; or else he subtracteth, T P. from the found number T L. that the remainer may be L P. the sine of the excesse of the third side: all which the three schemes following doe teach; in every of which, I will set downe an Example after* Birgius *his way.*

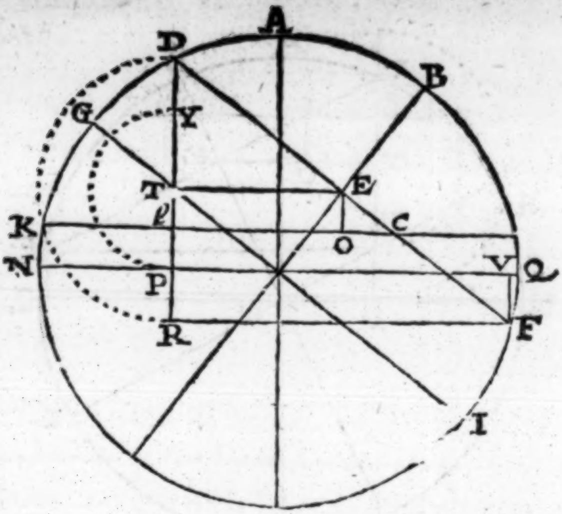
The



The first Example, which was the second of the second kind in the fourth Booke.

The lesser side G N, or Q I, —— 26. d. 20. m. } Adde and
The Comp: of the greater side GD, or F I, 30. 02. } Substract,
The summe is DN, 56. d. 22. m. The sine, 8325991. D P.
The difference F Q, 3. 42. The sine, 645323. P R.
the summe is, 8971314. r P.
The ½. is the first found number, 4485657. T P.
645323. D r.
A B C. 22. d. 35. m. 1 ¼. sec. is the arch of 3840334. D T, the ½ (of the right line,
50 d. 04. m. o. sec. put for A B C, as it were the lesser side.
72. 39. 1 ½. the summe thereof, the sine is 9545031.
27. 28. 58 ½. their differences; the sine is, 4614841.
Their difference is, 4930190.
The second number found is the ½. thereof, 2465095. CO,
or L T, the first number found, was —— 4485657. T P,
the totall is, 6950752. L P,
the sine of the complement of the third side.

The



The second Example, which was the third of the third kind In the fourth Booke.

The lesser side G N, or Q I, ——— 45 deg. 58 min.
The Compl. of the greater side G D, or F I, 30. 02.

The summe is D N, 76. —

Summe is D N, 76 deg. 00 m. the sine 9702957, D P.
The diff. F Q, is 15. 56. the sine 2745187, P R, or D Y.

The difference is 6957770, P Y. (numb.
The ½. is ——— 3478885. T P, the 1 found
2745187, P R, or D Y

38 deg. 29 min. 31 sec. 6224072, D T, ½. of the (right line.

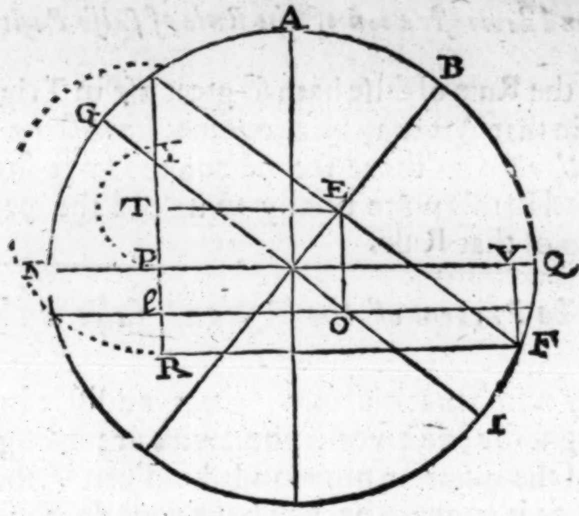
The lesser side — 51 deg. 30 min. 29 sec.
The excesse A B C, 22. 35. 00.

The summe is, 74. 05. 29. The sine 9617001
The difference, 28. 55. 29. The sine 4836600

The difference 4780401. (number.
The ½. is ——— 2390200. E O, or L T, the 2d. found
3478885. T P, the 1. found number.

The difference is, 1088685. L P, being the sine of the Complement of the third side.

The



The third Example, which was the fourth of the third kind in the fourth Booke.

The lesser side G N, or Q I. ——————————— 45. d. 58. m.

The Comp: of the greater side G D, or F I, 30. 02.

The summe is D N. 76. ——— The sine, 9702957 D P.

The difference Q F, 15. d. 56. m. the sine, 2745187. PR. or D Y.

the difference 6957770 Y P. (numb.

the ½. is ——— 3478885. T P. the 1, found

2745187. PR. to D Y.

38. d. 29. m. 31. sec. 6224072. TR. the ½. of

(the right line.

A B C, at the lesser side 10. ———————

The summe 48. d. 29. m. 31. sec. the sine 7488625

The difference 28. 29. 31. the sine 4770352

The summe 12258977

The ½. is the second found numb. 6129488. EO or TL.

the first found, numb. 3478885. TP.

The difference 2650603. P L. the sine of the excesse of the third side.

This is Bergius *his way; to speake of other wayes, it is not worth the labour.*

An addition to the fifth Booke containing the explaining and demonstration of the Rule of false Position.

Because the Rule of false hath so great use in Trigonometria, as a Scholler in that Art may be altogether freed from the intricate Rules of *Algebra*, as in the second booke I have shewed; I have thought good in this place briefly to unfold the precept and demonstration of that Rule.

The Precept of the Rule of false is thus.

Take any numbers at pleasure, great or little in stead of the number sought for; and worke therewith according to the order or Nature of the question propounded: Then if the facit or answer be just as it ought to be, you have your desire: But if otherwise. Note the difference or error by + or − And by another position either greater or lesser, then the first; repeat the former worke, and likewise note the errour by the said more or lesser, after multiply alternately by a crosse, the first position by the second error, and also the second position to the first error. Then if the Errors have like sines, subtract the lesser Product from the greater, and likewise the two errors the one from the other: But if the two errors have unlike sines, adde the two products together, and also the two errors.

And lastly, divide the totall or the remainder of the two products by the totall or remainder of the two Errors, the quotient is the true number sought for.

Therefore in this Rule there are three cases.

1. The first where both the errors are +.
2. The second where both the errors are −.
3. Third where the one is + and the other lesse.

Example.

What number is that to which if I adde $\frac{1}{3}$. thereof, and from the totall subtract $\frac{1}{2}$. of the whole the remainder is 100.

The true number sought is 90. as appeareth by the worke.

90. the number supposed.
30. the ⅓. thereof.
120. the summe added,
020. the ⅙ subtracted from thence,
100. Remaineth.

But imagine I know not the number sought for.

1 And for the first case. I will first suppose that number to be 144. And after I will suppose it, to be 108. As appeareth by the worke following:

1 *Position,*	144		2 *Position*	108
⅓	48		⅓	36
	192			144
⅙.	32		⅙.	24
	160	*The facits*		120
	100	*The true facit*		100
1 *Error* †	60	*The 2. Error* †		020
2 *Position*	108	*The 1. Position*		144
	6480	*The Product*		2880
	2880			
Resteth	3600	*the dividend. The 1. Error* † 60		
	40	*the divisor. The 2. Error* † 20		
	90	the quotient *Resteth* — 40 *The divisor*		
		or number sought for.		

For the second case I will suppose the number sought for; First 54. and after 72. As by the worke following is manifest,

1 *Position*	54	2 *Position*	72
⅓.	18	⅓.	24
	72		96
⅙.	12	⅙.	16
	60		80
	100	The true facit	100

1 *Error* — 40 2 *Error* — 20
2 *Position* -- 72 1 *Position* -- 54

2880 1080
1080

1800. the diuidend. 1 *Error* — 40
20. the diuisor. 2 *Error* — 20

(90. the quotient *the diuisor.* 20
or number sought for.

3 For the third case. I will first suppose the number required to be 54. And after 144.

The worke.

1 *Position* 54 2 *Position* — 144
$\frac{1}{3}$ 18 $\frac{1}{3}$ 48

72 192
$\frac{1}{6}$ 12 $\frac{1}{6}$ 32

60 160
100 100

1 *Error* — 40 2 *Error* + — 60
2 *Position* 144 1 *Position* — 54

5760 3240
3240

9000 the diuidend. 1 *Error* — 40
100. the diuisor. 2 *Error* + 60

100. diuisor

(90. the quotient
or number sought for.

I suppose you vnderstand the meaning of the rule of false. Now take the Demonstration, the ground of which is thus: That the *Errors* or falsities of the Positions: And of the Numbers found are proportionall one to another: That is, as the error of the first position is to the error of the second position; So is the error of the first found number, to the error of the second found number: I call the error of Positions, the excesse or defect of the Numbers supposed, abone or vnder the true number sought for. As in the first case.

1 *Position*	144	2 *Position*	108.
The true number	090	*The true number*	90.
The difference	054	*The difference*	18.

Therefore the error of the first position was: 54.
And of the second 18.

The errors of the Numbers found I call the excesse or want of the Numbers by the worke produced, either more or lesse then the Number to be produced: As in the first case.

1 Found number 160.	2 Found number	120.
100.	The number to be produced	100.
60.	The differences	20.

Then the error of the first found number was 60. And of the second 20. Therefore as 54. to 18, So is 60. to 20. The reason, Because the worke in both positions was after the same manner: that is by adding to the first supposed number $\frac{1}{2}$. thereof, and (from that totall) by taking away $\frac{1}{4}$. thereof.

The effect is answerable to the reason as by the worke following.

As 54. to 18. So 60. to 20.

Multiply 18.
by 60.

The Product is is 1080. which diuided by 54. the quotient is 20.

Because therefore: As the error of the first position to the error of the second position: So is the error of the first found number, to the error of the second found number.

Therefore if those errors be multiplied alternately, or by the Crosse, that is the errour of the first position, by the error of the second found number; And the error of the second position, by the error of the first found number: The Product of those two multiplications shall be equall. For if there bee foure numbers proportionall, the product of the two meanes shall bee equall to to the product of the two extreames, as was demonstrated in lines in the 1. Booke 42 Proposition & the same reason is in Numbers.

Seeing therefore the proposition is.
As 54. to 18. So is 60. to 20.

The product of the number 18. by 60. shall be the same with the product of 48. by 20. which the worke following shall make manifest.

The error of the first position		54
The error of the second position		18
The error of the second number found		20
The error of the first found	60	
	1080	1080

Moreouer in multiplication, it is all one whether I multiply the whole number by the whole, or one whole number by the parts of another: As for example. It is all one whether I multiply 7 by 7; or 7. by 4. and 3. For by both multiplications I shall finde 49. as reason teacheth, and the worke following sheweth.

7	7	7
7	4	3
49	28	21
	Add 21	
Fa:	49	

Therefore in the Example propounded in the first case: If I multiply the first position 144. by the error of the second number found, viz. by 20. It is all one as if I should haue multiplied 90. & 54 by 20. And consequently the product of the multiplication of the numbers 144. by 20. containeth the true number 90. 20. times: and the error 54 likewise 20. times.

2 In

1 In like manner, if I multiply the second position 108. by the error of the first found number, that is, 60. it is as much as If I should multiply 90. and 18. by 60. And consequently the product of the multiplication of the number 108. by 60. containeth the true number 90. 60 times; and also the error 18. 60. times.

But 18. taken 60. times, and 54. 20. times are equall, as before was demonstrated. Therefore, if from the product of 108. multiplyed by 60. I subtract the product of 144. multiplied by 20. I then shall subtract the error altogether, that came out with the first product: And also I shall subtract the true number 20. times: And the Remainder shall containe the true number 40. times, that is, as many times as the Remainder shall bee after the subducting of the error 20 from the error 60.

Therefore, if I divide that which remaineth after the subtraction of the one product from the other, by the Remainder of the errors of the two numbers found, one error beeing subtracted from an other; The Quotient of necessitie must bee the true number.

2 Againe, after the same manner in the second case: if I multiply the first position 54. by the second error 20, it is as much as if I should multiply 20, by 90. lesse 36. Then the product of 54. by 20. containeth 90. the true number 20, times, lesse by the error 36. 20. times.

And in like manner, if I multiply the second position 72 by the first error of the number found, viz, 40. It is as much as if I should multiply 90. lesse 18 by 40. Then the product of 72, by 40. containeth the true number 90. 40. times: lesse also by 40. times 18. the error or difference from the true number 90. Againe, 40. times 18. and 20. times 36. are equivalent as aforesaid; Therefore if I subtract the product of 54. by 20. from the product of 72. by 40. I wholly subtract the error produced in the first product; and also the true number 20. times. Then the remainder shall bee the true number 20, times. As the Error 20. subtracted from 40. there remaineth 20.

Therfore if I divide the remainder of the two Products by the remainder of the two errors of the numbers found: The quotient shall be the true number demanded.

3 In the third case, if I multiply the first position 54. by the error of the second found number, viz. 60, It is the same, as if I should multiply 90. lesse 36. by 60 then the product shall containe 90. the true number 60. times, lesse by 60. times the error 36.

If I multiply the second position 144 by 40. it is all one, as if I should multiply 90. and 54. by 40. therefore the product shall containe the true number 90. 40. times, and also the error 54. 40. times.

But 60. times 36. and 40. times 54. are equall in power one to another as afore: And therefore what is wanting in the one place, is ouer in the other: and consequently, if I adde the product of 54. by 60. to the product of 144. by 40. the totall shall be no more a falsе number, but shall containe 90, the true number, 60. times, and 40. times; that is a hundred times.

Therefore, if I diuide the totall of the Products, by the totall of the errors of the found numbers, I shall haue the true number required.

The end of the fifth Booke.

FINIS.

QVES-

QVESTIONS OF NAVIGATION, PERFORMED Arithmetically by the Doctrine of *Triangles*, without Globe, Sphære, or Map.

Written by RALPH HANDSON.

Wherein is manifested, *The disagreement betwixt the ordinary* Sea-Chart, *and the* Globe; *And the agreement betwixt the* Globe, *and a true* Sea-Chart: *Made after* MERCATORS *way, or Mr.* EDVV: WRIGHTS *projection: whereby the excellency of the Art of* Triangles *will be the more Perspicuous.*

THe Meridians in the ordinary Sea-Chart are right lines, all paralell one to another, and consequently doe never meet: Yet they cut the Equinoctiall and all circles of Latitude or Paralels thereunto at right angles, as in the Terrestiall Globe; but herein it differeth from the Globe: for that here, all the Paralels to the Equinoctiall being lesser Circles, are made equall to the Equinoctiall it selfe, being a great Circle, and consequently the Degrees of those paralels or lesser circles, are equall to the degrees of the Equinoctiall, or any other great Circle, which is meerly false, and contrary to the nature of the Globe, as shall bee hereafter more plainly demonstrated.

The

The Meridians in the terrestriall Globe, doe all meet in the Poles of the world, cutting the Equinoctiall, and consequently all Circles of Latitude or Paralels to the Equinoctiall at right Sphæricall angles; So that all such Paralels, doe grow lesser towards either Pole, decreasing from the Equinoctiall Line. As for example: 360. deg. or the whole Circle of the Paralell of 60. deg. is but 180. deg. of the Equinoctiall, and so of the rest; whereas in the ordinary Chart, that Paralell and all other are made equall one to another, and to the Equinoctiall Circle, as before said.

The Meridians in a Mappe of Master *Wrights* projection, are right Lines all paralell one to another, and crosse the Equinoctiall, and all Circles of Latitude at right angles, as in the ordinary Chart: but here though the Circles of Latitude are all equall to the Equinoctiall, and one to another, both wholly and in their parts or degrees; yet they keepe the same proportion one to another, and to the Meridian it self, by reason of the inlarging thereof, as the same Paralels in the Globe doe: wherein it differeth from the ordinary Chart. For that there the degrees of the Meridian, and the degrees of all Circles of Latitude are equall: And heere, though the degrees, of all Circles of Latitude are equall, yet are the degrees of the Meririan vnequall, being inlarged from the Equinoctiall towards either Pole to retaine the same proportion as they doe in the Globe it selfe: For as two degrees of the Paralell of 60. is but one degr. of the Equinoctiall or of any great Circle vpon the Globe: So heere, two degrees of the Equinoctiall or of any great Circle of Latitude, is but equall to one degree of the Meridian betwixt the Paralels 59. $\frac{1}{2}$. and 60 $\frac{1}{2}$. and so forth of the rest.

Also their agreement may be thus farther manifested:

Such proportion as one Circle hath to another; such proportion haue their Degrees, Semidiameters, and Sines, of like Arches one to another.

And therefore the proportion betwixt the Meridian and a Paralell, or betwixt a degree of the Meridian, and a degree of that Paralell, is as betwixt their Semidiameters.

So that if the Semidiameter of the Meridian be taken for the Radius, then the Semidiameter of any paralell, will be equall to the

the sine of the Complement of that Paralels distance from the Equinoctiall, in the like knowne parts as the Radius was of.

And therefore,

As the Radius, to the sine of the Complement of the latitude, or of that Paralels distance from the Equinoctiall: So is the Semidiameter of the Meridian, in knowne parts, to the Semidiameter of that Paralell in like knowne parts.

Or by changing of the middle tearme:

As the Radius to the Semidiameter of the Meridian: So is the sine of the Complement of the Latitude, to the Semidiameter of that Paralell.

Now every paralell in this projection, being equall to the Equinoctiall and consequently the degrees of every paralell beeing also equall to the degrees of the Equinoctiall; the Meridian, and the degrees thereof, must of necessitie be inlarged, and increase from the Equinoctiall towards either Pole; to retaine the same proportion that is betwixt the Meridian, and the paralels of the Globe.

For if the sine of the Complement (or the Semidiameter of any paralell) which is alwayes lesse then the Radius or Semidiameter of the Meridian, be made equall to the Radius or that Semidiameter: Then that Radius or Semidiamter of the Paralell, shall haue such proportion to the secant of that paralels distance from the Equinoctiall, as the sine of the Complement should haue had to the Radius; because

The Radius is a meane proportionall Number betwixt the sine of the Complement of any Arch, and the Secant of that Arch.

And therefore as the sine of the Complement, is to the Radius: So is the Radius to the Secant of the arch giuen. And contrarily,

As for Example.

If I would know the proportion betwixt the Meridian, and the Paralell of 50. deg. or betwixt a degree of the Meridian, and a degree of that paralell in minutes or miles; I say according to the proportion of the Globe.

As 10000. the Radius to 6417. the sine of 40. deg. (being the Complement of 50, deg. the paralell giuen.) So is 60. minutes or miles, answering to a degree of the Meridian; to 38.$\frac{576}{}$. minutes,

minutes or miles, answering to a degree in the paralell of 50. deg. Or, if I had said, according to Mr. *Wrights* projection:

As 15557, (being the secant of the Latitude (is to 10000. so is 60, deg. to 38 m. $\frac{3144}{15557}$. it had been all one with the former worke.

The reason heereof is, that if you haue three Numbers in continuall proportion, that is, as the first to the second; So is that second to the third; you may by hauing any two of them (so the second be one) and a third Number giuen, find a fourth Number, in such proportion to the third as the second was to the first, As for example: Let 4. 6, and 9. be three numbers in continuall proportion, and 12. be another Number giuen. Then you may say as 4. to 6, so is 12. to 18. Or

As 6. to 9. so is 12. to 18. because 6. is a meane proportionall Number betwixt 4. and 9.

In like manner. If I am to say, as the sine of the Complement to the Radius; I may say, as the Radius to the secant of the Arch giuen, and whatsoeuer number shall bee giuen for the third, the answer will be still one and the same.

But of the proportion that is held in the inlarging of the degrees of the Meridian from the Equinoctiall towards either Pole, Mr. *Wright* himselfe hath demonstrated the same in the errors of Nauigation by the Tables of Latitude, which he hath calculated by the continuall addition of the Secants: where you may more amply satisfie your selfe touching that argument,

Now followeth the Questions themselues to be performed Arithmetically: viz.

1 By the Latitudes and Longitudes of two places giuen to finde the Rumbe or point of the Compasse of bearing: and their Rumbe distance.

2 By the distance and Latitudes of two places giuen, with the Longitude of one of them: to find their Rumbe, difference of Longitude, and the longitude of the other place.

3 By the Rumbe and Latitudes of two places giuen, with the Longitude of the one place: to finde their distance, difference of Longitude, and the Longitude of the other place.

4 By the Longitudes Rumbe, and one Latitude giuen, to finde the other Latitude and their distance.

5 By

5 By the Rumbe, the distance and one latitude and longitude given; to finde the other latitude, and their difference of longitude, and consequently the other longitude.

For the better and more easie understanding the resolution of these or the like questions, It is first necessary to know of two places given, whether lyeth more Southerly, or Northerly, Easterly, or Westerly.

All latitude on the terrestiall Globe is accompted from the Equinoctiall towards either Pole, being numbred in the Meridian from 1. to 90. deg. and taketh the denomination according to the pole, towards which it is numbred; that is, either Northwards or Southwards. And therefore if both places lye on the North-side of the Equinoctiall, the lesser latitude lyeth more Southerly, and the greater latitude lyeth more Northerly, the difference of latitude being the remainder of those two numbers when the lesser latitude is taken out of the greater.

And contrarily, if both places lye on the South side of the Equinoctiall, the greater latitude lyeth more Southerly, and the lesser more Northerly; the difference of latitude being found as before.

If one place lye under the Equinoctiall, and the other without it, that without the Equinoctiall lyeth more Northerly or Southerly according to the denomination of the latitude of that place the difference of latitude being the latitude given.

And lastly, if one place lye on the North, and the other on the South side of the Equinoctiall; that on the South side lieth more Southerly; And that on the North side more Northerly: the difference of latitude being the summe of both latitudes added together.

Againe, all Longitude on the terrestriall Globe, is accompted from some fixed Meridian into the East, being numbred in the Equinoctiall or some Circle paralell unto it, from 1. to 360. degrees. And therefore of two places given, differing in Longitude, the greater longitude lyeth more Easterly, and the lesser longitude lyeth more Westerly; except (accompting from the lesser to the greater longitude) they are more then 180, degrees distant, for then the lesser longitude lyeth more Easterly and the greater longitude lyeth more Westerly: the difference in lon-

longitude being the remainder when the lesser number is taken out of the greater; but if that remainder exceed 180. deg. then that excesse taken from 360. deg. leaveth the difference of longitude.

Hereby it appeareth that the limits or bounds of North and South are the Poles themselves: but of East and West there are no limits.

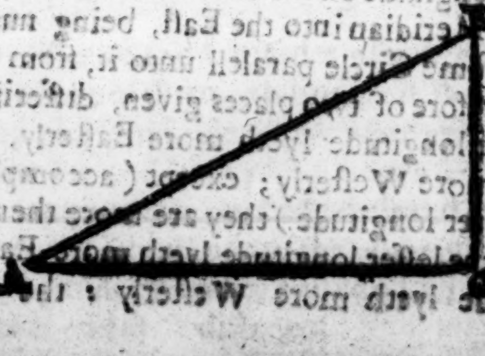
PRO: 1. *To find how many miles altereth a degree of the Meridian; or serveth to raise or depresse the Pole one degree upon any Rumbe or point of the Compasse given.*

The segment of the Meridian, and the segment of any Rumbe included betwixt any two Paralels of like distance, are in one and the same proportion one to another in all Latitudes. And therefore by the first Axiom of the third of *Pitisc.* As the Radius is to the secant of the angle, included betwixt the Meridian and the Rumbe given: So is the miles or minutes answering to a degree of the Meridian betwixt any two paralels; to the miles or minutes altering a degree of Latitude (or raising or depressing of the Pole one degree) upon the Rumbe or point of the Compasse given.

Or else I might say. As the sine of the angle differing from the Paralell is to the Radius; So is the miles or minutes of one degree of the Meridian: to the miles or minutes that I am to saile upon that point, to alter one degree of the latitude.

Example.

I demand how many Miles I shall saile to alter one degree of Latitude upon an E. N E. W. N W. E. S E. or W. S W. Rumbe?



In the triangle A B C. Let B C, represent 60. m. or a degree of the Meridan: A B C, the angle giuen different from the Meridian, either East or West; whose Complement is the angle B A C, from the Paralell or East and West line, eyther to the Northward or Southwards, that is A B C. is an angle of 6. points of the compasse, or 67 deg. 30 m. from the Meridian and B A C, is an angle of two points, or 22. deg. 30. m. from the Paralell, accompting for euery point of the Compasse, 11. deg. 15. min. And lastly, let the angle A C B, be an angle of 90. deg. or a right angle: because the Meridian and the Paralell cut one another at right angles, A B. representing the Paralell, or East and West line, and let *AC*, be the line sought for, Then I say.

As C B, *the* Radius 10000. *to* B *A.* 26131. *the* Secant *of the angle* *A*BC, 67. *d.* 30. *m. So is C B,* 60. *Miles or one degree of the Meridian: to B A.* 156. $\frac{7860}{10000}$. *miles that I shall saile vpon that point of the Compasse, to alter one degree of Latitude: Or,*

As B C, 3826. *the sine of the angle B A C,* 22 *d.* 30 *m. to* 10000 *the Radius B A, So is B C.* 60. *miles: to B A,* 156. $\frac{3164}{3826}$. *miles: as before.*

And this Rule, is generally held, aswell upon the ordinary Chart, as on the Globe, or a Map made after M. *Wrights* projection, Wherein you are to note, that if the course be Northerly, you shall raise or elevate the Pole: And contrariwise if the course be Southerly, you shall lay or depresse the Pole, in the North Latitude. But if you be to the Southward of the Equinoctiall, and your course Southerly, you shall raise the Pole: and depresse it when your course is Northerly.

Prop. 2. *To finde the distance betwixt two places lying in a Paralell, that is East and West, one from another: their longitudes and latitude being given.*

Multiply their difference in Longitude by the miles answering to a degree of Longitude in that Paralell; the Product will be the distance required: Or else,

Mul-

Multiply their difference in Longitude by 60. miles, answering to a degree of the Equinoctiall : And then either,

As the Radius to the sine of the Complement of the Latitude giuen : So is the difference of longitude multiplyed by 60. to the distance required. Or,

As the Secant *of the Latitude to the* Radius *: So is the difference of Longitude multiplyed by 60. to the distance in miles, as before.*

Example.

Let the Southermost part of the Island of S. *Maries*, one of the Azores : and Cape S. Vincent, both lying in the latitude of 37 d: be two places whose distance is required. And admit their longitudes to be as followeth.

S. *Maries Iland Longitude* ————	351. *deg.*	10. *m.*	
Cape S. Vincent Longitude ————	007.	10,	Which
Subtracted, The Remainder is ————	344	————	Which
Againe subtracted from 360 *deg, resteth* ——	16		*For*

their difference of longitude : which multiplyed by 47. $\frac{9160}{10000}$. Miles, being the Miles answering to 1. degree in the Paralell, of 37 degrees, as is before taught, produceth 766. $\frac{6560}{10000}$. Miles, for their Paralell distance : Or,

As the Radius 10000. *to* 7986, *the sine of the Complement of* 37, *deg the Latitude giuen : so is* 960. *the difference of Longitude multiplyed by* 60. *to* 766, $\frac{8560}{10000}$. *miles the distance required : Or,*

As 12521. *the* Secant *of* 37. *deg. being the Latitude giuen to* 10000, *the Radius : So is* 960. *the difference of longitude multiplyed by* 60. *To* 766, $\frac{[illegible]}{12521}$. *miles, the distance as before.* All which workes doe agree whereas in the ordinary Chart, their distance will be found to be 960. miles, which is more then the truth, by almost 194. miles, and would haue differed much more, if the two places giuen had beene further distant from the Equinoctiall line. But if it bee demanded, whether of the two places lye more Easterly ; by the rules aforesaid, Cape S. Vincent is found lye more Easterly then S. Maries Island, by so much as their difference of longitude is.

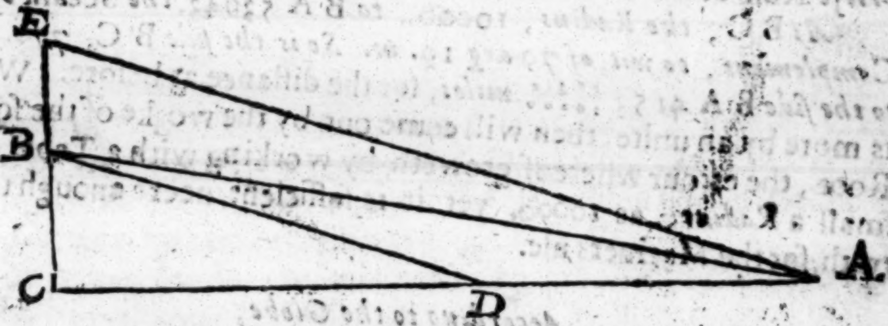
Pro-

Pro. 13. *The Longitudes and Latitudes of two places being given both on the one side of the Equinoctiall, to find their bearing and Distances.*

Let the *Lizard* in *Cornwall*, and an Island lying in the mouth of *Lumleys* Inlet, in *fretum Davies*, bee the two Places given, and let their Course and Distance bee required: Admit the Latitude and Longitude of those two places, to be as followeth.

Lumleys Inlet, Latitude North, 63 deg. 00 m. Longit. 309 deg.
The *Lizard*, Latitude North, 50. — 10. — Longit. 17.

Their difference in Latitude — 12 deg. 50 m. differ. Long. 68.
Which is 770 min. for the difference of Latitude, and their difference of Longitude is 4080 min. both their differences of Latitude and Longitude, being multiplyed by 60 min. as is usuall upon the ordinary Chart, according to which we will first worke.



According to the ordinary Chart.

In the right angled Triangle A B C, let B represent *Lumleyes* Inlet; and A, the Lizard point: in which Triangle according to the ordinary Chart, are the two lines including the right angle B C A given, together with the right angle B C A, to wit, B C 770 minutes, the difference of Latitude: And C A, 4080 min. the difference of Longitude; Then I say by the second Cons: of the first Axiome of the 3. Booke of *Pitiscus*.

O

As

As BC, 770. *min. the difference of Latitude to* C A, 4080. *min.* the difference of Longitude, *So is the Radius* 10000 *to* 52987, *the* Tangent *of* 79. *deg,* 19. *min. for the acute angle ABC.* Whose Complement is 10. deg. 41. min. For the other acute angle B A C.

Whereby I conclude the bearing of *Lumleys* Inlet from the *Lizard* to be 10 deg. 41. m. from the West Northwards, that is almost W. by North. And the bearing off the Lizard from *Lumleys* Inlet to be 79 deg. 19 min. from the South Eastwards, that is, E, by S. and 34. to the Eastward.

Now for their Distance, you may find it by the square Root, by extracting the square Roote out of the summe of the two squares of the sides given, or else by the second Axiome of the 3. of *Pitiscus:* Thus,

As B C. 1854, *the sine of the angle* B A C, 10. *deg.* 41, *min. to B A, the Radius,* 10000. *So is the side B C,* 770 *miles to the side B A.* 4153 $\frac{[illegible]}{1854}$ *miles for the Distance required, Or by the second briefe* Rule of the 5. Booke.

As B C, *the Radius*, 10000, *to* B A 53943, *the* Secant *of the Complement, to wit, of* 79 *deg* 19. *m. So is the side* B C, 770. *min. to the side* B A 4153 $\frac{6111}{10000}$. *miles,* for the distance as before. Which is more by an unite, then will come out by the worke of the square Root, the errour whereof groweth by working with a Table of so small a *Radius*, as 10000, yet it is sufficient neere enough to the truth, for the Mariners use.

According to the Globe,

When B C is given 770. min. for the difference of Latitude upon the Meridian, and *C A,* 68. deg, or 4080. m. for the difference of Longitude in the middle Paralell betwixt those two places, the same line C A, must be fore-shortened in such proportion as the *Radius* is to the sine of the Complement of the middle parallel, by finding the sine of the complement of the middle Paralell, in this manner. By the 2. *Pro*: I say,

As 10000. *the Radius to* 4540. the sine of the Complement of 63. deg. to wit. 27. deg. *So is* 4680. to 1852. the miles answering to the difference of Longitude in the paralell of 63. degr. Againe I say:

As 10000. *the Radius*, *to* 6406 the sine of the Complement of 50. deg. 10. m. to wit, 39. deg. 50. min. So is 4080. to 2613. the miles answering to the difference of Longitude in the Paralell of 50. deg. 10. min. So is found.

The miles answering to the difference of Longitude in the paralell of 63. deg. —— } 1852. Miles.

And in the Paralell of —50. deg, 10. m. is 2613. Miles.

The summe whereof is ——4465.

½. thereof is —— 2232. ½. for the line CD, representing now the difference of Longitude in the Triangle BCD. Or to find the line CD, more briefly at one worke.

The sine of the complement of 63. deg. to wit, of 27. d. is 6406

The totall —— 10946

½. Whereof is —— 5473

Then I say. *As* 10000: *the Radius, to* 5473. taken heere for the sine of the Complement of the middle Parallell; So is 4080, to 2232. for the line C D, as before.

Now in the right angled triangle B C D, I have the two sides given, comprehending the right angle, to wit, the side B C. 770. m. and the side C D, 2232. Wherefore I say: As B C, 770. to C D, 2232. so is *B C*, the *Radius* 10000. to *C D*, 28987. the *Tangent* of 70. deg. 58. m. for the acute angle *C B D*, whose Complement is 19 deg. 2, min. for the other acute angle *B D C*.

Yet I may worke (by compounding the Proportions) more briefly. For whereas I said before,

As 10000. to 5473. So is 4080. to another Number: and As 770. to the other Number: So is 10000. to the Number sought. For I may by omitting the two *Radij* say,

As 770, to 5473. So is 4080. to 28999. the *Tangent* of 79. deg. 58. min. for the acute angle *CBD*, as before.

So that the bearing off the Lizard from *Lumleys* Inlet, is hereby found to be 79. deg. 58. min. from the South Eastwards: which dividing by 11. deg. 15. min. is 6 points 3. d. 28. m. that is E, S E. and 3. deg. 28. min, towards the East; and the bearing of *Lumleys*

Inlet from the Lizard is West by N. and 7 deg. 47 min. towards the North.

Now having the three angles and the two sides comprehending the right angle in the Triangle B C D; to wit, B C, 770. miles. and C D 2232. miles, I may find the third side B D, as was taught according to the plaine Chart, in the former part of this Proposition: *viz.* eyther by the extracting the square Roote out of the summe of the squares of the two sides; or by the second briefe Rule, of the fifth Booke of *Pitiscus*. For,

As 10000. *the Radius, to* 30664. *the Secant of the angle* C B D 70. *deg.* 58 *min. So is the side* B C, 770 *miles, to the side* B D, 2361. *miles, for the distance sought for.*

Or else,

As B C, 3261. *the Sine of* 19 *deg.* 2. *min. is to* B D, *the Radius,* 10000. *So is the side* B C, 770. *miles, to the side* B D, 2361. *miles, for the distance as before.*

According to the true Sea Chart.

But suppose C A, the difference of Longitude to bee 4080. miles of the Equinoctiall, or of any Paralell equall unto it, as all the paralels are equall thereunto in a Chart after Mr. *Wrights* projection: then cannot the line C B, represent the true difference of Latitude, but must bee inlarged according to the proportion that is betwixt the Equinoctiall, and the middle paralell betwixt the Latitudes given, which although it bee not precisely true according to Art; for that the Sines, Tangents, and Secants, doe not differ by equall proportion, yet it is sufficient neere enough for the Mariners use, and such as have not Mr. *VVrights* Booke, to take this way.

And it is thus performed.

First, adde the Secants of both the Latitudes given, and of that Number take the halfe.

Then say,

As the Radius to that halfe which is here taken for the Secant of the middle Paralell. So is the difference of Latitude in equall parts given, to another number in like equall parts, which sheweth the line C E.

As

As for Example.

The Secant of 63 deg. —— is, —————— 22027
The Secant of 50 —— 10 min. is, —————— 15611
Which added together, maketh —————— 37638
½ whereof is —————— 18819 for the Secant of the middle Paralell.

Then say; *As the Radius* 10000. *to* 18819. *So is* 770. *the difference of Latitude, when* C D *is taken for the difference of Longitude*: So have I another Triangle A E C, which is equiangled to the Triangle A B C; and therefore their sides are proportionall by the 46 of the first booke of *Pitiscus*. And by this way to find their bearing and distance. I say,

As the side A C, 4080. *m. to the side* C E, 1449. *So is the Radius* 10000. *to* 3551. the Tangent of 19 deg. 33 min. for the acute angle E A C, whose Complement is 70 deg. 27 min. for the other acute angle C E A.

And for their distance it is found as is before set downe to bee 2301. miles.

Yet their bearing and distance, may be found more exactly then by any of the former workes, by the helpe of the Table of Latitudes, calculated by Mr. *Wright* as afore said, after this manner.

Take the difference of the Meridionall parts answering to the Latitudes given, for the side C E, and for the other side A C, take the difference of Longitude in miles, and multiply that by 10. Then say, as C E, the difference of Latitude in equall parts is to C A, *the difference of Longitude in miles, multiply by* 10. *So is the Radius to the Tangent of the angle* A E C.

As for Example.

The Meridionall parts in that table for 63 deg. —— is 49053
And for —— 50 deg. 10 m. is 34901
The difference is —— 14152. for the side E C. Againe, the difference of Longitude in miles, is 4080. which multiplyed by 10. maketh 40800. for the side A C.

Then I say: As E C 14152. to A C 40800. So is E C 10000. the Radius. To A C 28829. the Tangent of 70 deg. 53 min. for

for the acute angle A E C, whose Complement is 19. degr. 7. min. for the other acute angle C A E. Then to find their distance, I say, as before taught.

As C E, the Radius 10000. to E A, the Secant of 70. deg. 53. min. to wit, 30535. So is C E, 770. the difference of the Latitude in miles to E A, 2351. miles, for the true distance upon that Rumbe.

Now let us compare these works together, and see their difference, taking the last work for the truth, because it is wrought by Tables calculated, to every Minute of the Meridian, whereas the former workes are wrought, without the help of those Tables.

By the ordinary Chart, the bearing off the Lizard from *Lumleys* Inlet. is 79. d. 19. min. from the S. Eastw. the distance 4153.
the true bearing 70. 53. m. from the S. Eastw : true distance 2351,
the diff. too much 08. 26. to the Eastw : distance too much 1802. (miles.

By working by the sine of the Complement of that middle Paralell, the bearing of the Lizard from *Lumleyes* Inlet is
70. deg, 58. m. from the S. Eastw, the distance 2361
the true bearing 70. 53. from the S. Eastw : the distance 2351
the diff : too much — 05. to the Eastw : distance too much 0010 (miles.

By working by the Secant of the middle Paralell, the bearing off the Lizard from *Lumleys* Inlet, is as followeth :
70. d. 27. m. from the S. Eastw : the distance 2301
true bearing, 70. 53. from the S. Eastw : the true distance 2351
The difference — 26. too little to the Eastw. the dist. too little 050 (miles;

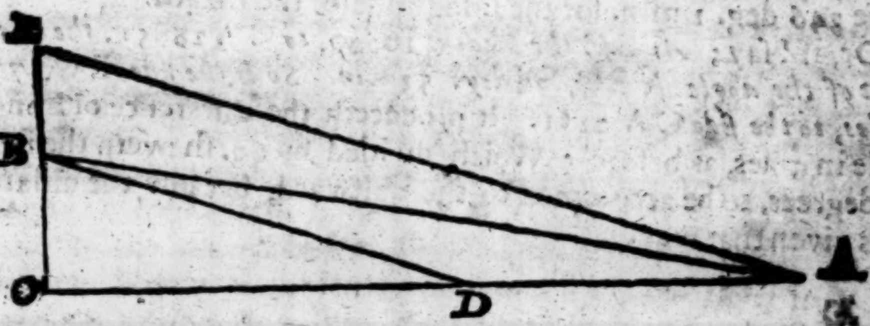
So that hereby it appeareth, that any of the former workes is sufficient neere enough for the Mariners use, onely the plaine Chart is to be rejected, for that it differeth from the truth in the bearing more then ¾. of a point of the Compasse. And in the distance it bringeth out too much by 1802. miles.

Pro. 4

Prop. 14. The Latitudes of two Places being given, with the Longitude of one of them, either Eastward or Westward, and their distance: To find their bearing and difference of Longitude. And thereby the Longitude of the second place.

Let *Lumleys* Inlet and the Lizard as afore be the two places given; And let the Latitude and Longitude of *Lumleys* Inlet bee given for the one place, and the latitude of the Lizard for the other place, together with their true distance 2351. miles.

Lumleys Inlet Latitude 63 degrees. and Longitude 309 degrees, the latitude of the Lizard 50 d. 10 m. North longitude, their distance to the Eastward is, 2351. miles.



According to the ordinary Chart.

In the Triangle A E C, let the side E C, be given 770 min. for the difference of latitude in the ordinary Chart, and let the side E A, 2351 miles in the same Triangle bee given for the distance; And let the acute angles A E C, and C A E, together with the line C A, be demanded. I say,

As C E 770 miles, to E A, 2351 miles. So is C E, the Radius 10000. to E A 30532. the Secant of 70 deg. 53 min. for the acute angle A E C, whose Complement is 19 deg. 7 min. for the other acute angle C A E. Then because the distance was given Eastward, I conclude the bearing of the Lizard from *Lumleys* Inlet, to be 70 deg. 53 min. from the South, Eastwards; And I say from the South, because the latitude of the Lizard is the lesser Latitude.

Againe, for the difference of Longitude, it may be found either by the square root, thus:

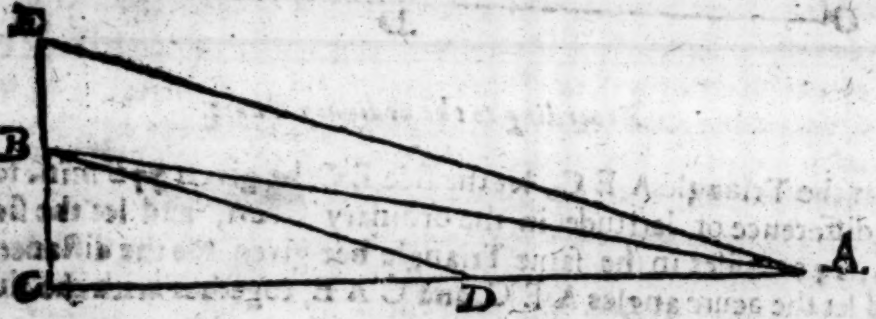
Subtract the Square of 770 out of the Square of 2351, the square Root of the Remainder is the difference of Longitude in miles, which divided by 60 sheweth the same in degrees:

As for Example.

The square of 2351. E A, is ——— 5527201. from whence
The square of 770. E C, ——— 592900. subtracted,

The remainder is the square of C A, 4934301. whose square Root is 2221 miles, for the side C A. Which divided by 60, giveth 37 deg. 01 min. for the difference of Longitude; So that if I adde 37 deg. 1 min. to 309 deg. the Longitude of *Lumleys* Inlet, I shall have 346 deg. 1 min. for the Longitude of the Lizard.

Or, if I say; *As* E C *the Radius* 10000, *to* C A 28851, *the Tangent of the angle* A E C, 70 *deg.* 53 *min. So is the side* E C, 770 *miles, to the side* C A 2221. It produceth the difference of Longitude in miles, as before: Which divided by 60. sheweth the same in degrees, to be accompted to the Eastward, because the distance was given that way.



According to the Globe.

Againe, In the Triangle B C D, let the side B C, bee given 770 miles for the difference of Latitude according to the Globe. And let B D 2351 miles in the same Triangle be given for the distance: And let the demand be as before.

I say, *As* B C, 770 *miles, to* B D 2351 *miles*: *So is* B C, *the Radius* 10000. *To* B D, 30532. *the Secant of* 70 *deg.* 53 *min.*

as

as before it was found by the plaine Chart. And likewise, for the difference of Longitude, it is found in Miles after the same way; by saying:

As B C, the Radius 10000. to C D 28851. the Tangent of the angle D B C, 70 deg. 53 min. So is the side B C 770 miles, to the side *C D*, 2221. miles, for their difference of Longitude in miles: But because these miles or minutes are answerable to so many minutes of the middle paralell betwixt the Latitudes given, in such proportion as is betwixt the Semidiameter of the middle paralell and the Equinoctiall: Therefore first, having found by the 3. *Pro.* the sine of the complement of the middle paralel to be 5473.

I say, *As* 5473. *to the Radius* 10000. *So is* 2221. *minutes of the middle Paralell to* 4058. *minutes of the Equinoctiall,* which divided dy 60. giveth 67. degrees 38. min. for their difference of Longitude: which 67 deg. 38 min. added to 309 deg. the Longitude given, the totall is 376 deg. 38. min. from whence 360 deg. be taken away, the Remainder is 16 deg. 38 min. for the Longitude of the Lizard.

The true Sea-Chart.

Againe, If you worke in the triangle A E C. according to Mr: *Wrights* projection, you shall finde the bearing and difference of Longitude in minutes (according to the plaine Chart) to agree with the worke there set downe. But to reduce the 2221 min. of Longitude of the middle paralell into minutes of the Meridian; you shall by the 3. *Proposition* find the Secant of the middle Paralell, which is there found to be 18819.

Then,

As the Radius 10000. to that Secant, So is 2221. of the middle Paralell to 4179. minutes of the Meridian: which 4179. devided by 60 min. giveth 69 deg. 39 min. for the difference of Longitude, whereby the Longitude of the Lizard is found to be 18. deg. 39. m.

Yet againe, more neere by the helpe of the Table of Latitudes, First, find the difference of the Meridionall parts, by the 3. *Pro.* which is 14152: Then after you have found the two acute angles as afore, *viz.* A E C, 70 deg. 53 min. and C A E, 19 deg. 7 min. you may say;

As 100000. E C, being the Radius multiplyed by 10. is to C A. 28830. the Tangent of the angle AE C, 70 deg. 53. m. So is C A, 14152. the difference of the Meridionall parts, to 4080. the side C A, for the minutes or miles of the difference of Longitude, which being divided by 60, giveth 68 deg. for the difference of Longitude, and thereby the Longitude of the Lizard is found to be 17 deg.

Thus you may perceive that in this question supposing the Latitudes and distance to be true; there is no difference in finding the Rumbe or bearing, in any of these three operations, either by the ordinary Chart, the Globe, or Map. The onely difference is in the Longitude. For

The true difference in Longitude by the ——— }
tables of Latitude, is, ——————————. } 68. deg.
By the ordinary Chart, the difference of Longitude is 37. deg.
which is too little by ———————— 31: deg.
By the sine of the Compl: of the middle Paralell dist. 67. 38: m.
which is too little by ———————— 00. 22.
By the Secant of the middle paralell the diff. in Longit. 69. 39.
which is to much by ———————— 01, 39.

Pro: 5 *The Latitudes of two places being given with the Longitude of one of them and their bearing; to finde their distance, difference of Longitude, and consequently the Longitude of the second place.*

Let the Lizard and *Lumleyes* Inlet be the two places given, as before: And let the Latitude and the Longitude of the Lizard, And the Latitude of *Lumleyes* Inlet, together with their bearing be given: And let their distance, &c. be required.

Lumleyes Inlet, Latitude North 63. d. ——— longitude ———
The Lizard, Latitude North — 50. 10: m. Longitude 17 deg.
The bearing off *Lumleyes* Inlet from the Lizard 19. deg. 7. min: from the West-Northward, that is 70. deg. 53. m. from the North Westwards.

Accor-

According to the ordinary Chart.

In the Triangle A E C, let the side E C be given 770 miles, for the difference of Latitude, together with the acute angles A B C, 70 deg. 53 min. and E A C, 19 deg. 7 min. And let the sides A B and A C, be sought for: First, I say,

As E C, 10000. *the Radius*, *to* E A, 30535. *the Secant of the angle* C E A. *So is the side* E C, 770. *min. to the side* E A, 2351. *miles for the distance required.* Which being had, their difference in Longitude is found by the 4 Pro: to bee 37. deg. 1 min. which for that the bearing is Westward, is to be taken from the Longitude given. *viz.* 17 deg. Now I cannot take 37 deg. 1. min from 17 deg. And therefore I take it from 360. deg. and 17. deg. that is 377. deg. the Remainder is 339. deg. 59. min. for the Longitude of *Lumleys* Inlet, being the thing required.

According to the Globe.

In the Triangle B C D. Let B C, represent according to the Glode difference in Latitude 770. min. and let the acute angle C B D. 70. deg. 53 min. and the acute angle B D C, 19. deg. 7. min. be also giuen for their bearing. And let the distance, &c. be demanded: First. I say;

As the Radius B C, 10000. *to* D B, 30535. *the Secant of the angle* C D B, 70. *deg.* 53 *min. So is the side* B C, 770 *min. to the side* B D 2351. *miles, for the Distance required.* And afterwards for the differeuce in Longitude. It is found by the 4. *Pro*: to bee 63. deg. 38 min. which subtracted from 360 deg. and 17. deg. that is from 377 deg. the remainder is 309 deg. 22 min. for the Longitude of *Lumleys* Inlet.

The true Sea-Chart.

Againe, In the triangle A E C, as before taught in the ordinary Chart, the distance will be found to be 2351. miles, and the difference of longitude by the 4. Pro: to be by working by the Secant of the middle paralell, 69 deg. 39 m. for the difference of longitude, which taken from 377 deg. the Remainder is 307. deg. 21 m. for the longitude of *Lumleys* Inlet.

But

But if you worke by the table of Latitudes as is set downe in the last *Pro:* the difference of the Longitude will be 68 deg. which taken from 377 deg. the Remainder is 309 deg. for the true Latitude of *Lumleyes* Inlet.

The difference of these workes is onely in the difference of longitude, as is set down in the last *Pro.* afore-going.

Pro: 6. The Longitude of two places being given together with their bearing, and one Latitude, to find the other latitude and their distance.

According to the ordinary Chart.

Let the Lizard and *Lumleyes* Inlet be the two places given as before: And let *A* in the Triangle *A E C.* be given for the longitude, and the Latitude of the Lizard, viz: Longitude 17 deg. and latitude 50. deg. 10. m. North: And in the same Triangle let E represent the longitude of *Lumleyes* Inlet 309. deg. whose latitude is sought for: And let the bearing of *Lumleyes* Inlet from the Lizard be given 19. deg. 7. min. from the West Northwards, that is 70 deg. 53 min, from the North Westwards; whereby is given, the two acute angles E A C. 19 deg. 17. min. and A E C. 70. deg. 53. min. Then have I given the Triangle AEC sufficient tearmes for the resolution thereof. viz: the difference of longitude 68. deg: or 4080. min. represented by the line A C. and the 3. angles, whereby I resolve that Triangle as followeth. First, according to the ordinary Chart, I say:

As A C the Radius 10000, to C E 3466. the Tangent of 19. deg. 7. m. So is A C, 4080. miles to C E. 1414. miles for the difference of the latitude, which divided by 60. giveth 23.d. 34. m. Now because the course was given to the Northwards, I adde 13. deg. 34. min. the difference of latitude found to 50. deg. 10. m. the latitude given, the summe is 73. deg. 44. m. for the latitude of *Lumlyes* Inlet. Again for the distance, I say.

As A C the Radius 10000 to A E 10583, the Secant of the angle C A E So is 4080, m. the difference of longitude to A E 4317. the distance required.

And

And because it is difficult in this proposition to find the difference of Latitude and distance by the bearing, and difference of Longitude, because onely one Latitude is given, whereby J can neither take the sine of the Complement or Secant of the middle paralell as was done in the former Propositions, I will therefore onely, in this, shew the true working of it by the helpe of the Tables of Latitude, thus;

First, find the number in the Tables of Latitude answering to the Latitude given: And then say, As the Radius to the Tangent of the angle different from the Paralell: So is the difference of Longitude multiplyed by 10. to another number, which added to the number answering to the Latitude of the place given (if the places Latitude sought for be more Northerly) giveth you the number answering in those Tables, to the Latitude of the second place: Or that number found, subtracted from the number answering to the Latitude of the place given, if the Latitude demanded be Southerly, leaveth the number answering in the same Tables to the Latitude of the second place.

And therefore.

As A C the Radius 10000, to C E 3466, the Tangent of 19 deg. 7 min. So is A C 40800. the difference of Longitude in minutes multiplied by 10. to C E 14141. which for that the bearing is given Northward, I adde to 34901. the number answering in the Tables of Latitude to 50 d. 10 m. the Latitude given, the sum is 49042, and that number I seeke in the Tables of Latitude, where I find it to answer to 67 degrees of Latitude: Then I say, the Latitude of *Lumleyes* Inlet is 63 deg. North, which was required.

The Distance is also found, having both Latitudes and Longitudes, by the third or fifth *Pro*: to be 2351 miles.

The difference of the ordinary Chart in this *Pro*: from the truth, is in the Latitude of the second place, and the distance.

For by the ordinary Chart.

The Latit. of *Lumleys* Inlet is 73 deg. 44 m. the distance 4317 min.
The true Latitude is ——— 63. ——— true distance 2351.

The difference is too much Latit. 10. d. 44. in distance 1966 miles, which is produced in the ordinary Chart more then the truth.

Pro: 7.

Pro: The Latitude and Longitude of one place being given, together with the Rumbe and distance to find the Latitude and Longitude of the second place.

Let E, in the Triangle A E C, represent *Lumleys* Inlet, whose Latitude is given 63 deg. North, and Longitude 309 deg.

And let *A* represent the *Lizard*, whose Latitude and Longitude is required; let the angle A E C, be given 70 deg. 53 min. for the bearing from the South Eastwards, and let the distance E A, be given 2351 miles.

According to the ordinary Chart.

First, I say to find E C, the difference of Latitude, and thereby the Latitude of the second place.

As A E 30535, *the Secant of the angle* A E C, 70 *deg.* 53 *min. to* E C, *the Radius* 10000. *So is* A E, 2351 *min. the Distance given, to* E C, 770 *min. the difference of Latitude: Or else by bringing the Radius into the first place,* by the fourth briefe Rule of the fifth of *Pitiscus.*

As A E 10000. *the Radius: To* E C 3274. *the sine of the Complement, to wit,* 19 *deg.* 7 *min. So is* A E 2351. *min. the Distance given to* E C, 770 *min. the difference of Latitude.* Which divided by 60 min. the Quotient is 12 deg. 50 min. which for that the bearing is given Southward, I subtract from 63 deg. the Latitude given, the Remainder is 50 deg. 10 min. for the Latitude of the *Lizard.*

Then by the 5. *Pro:* having the Latitudes and bearing given, together with one Longitude, you shall find the Difference of longitude by the ordinary Chart to bee 37 deg. 1 minute, which because the bearing is Eastward, is to bee added to 309. the Longitude given, the Totall is 346 deg. 1 minute, for the Longitude of the *Lizard.*

According to the Globe, or true Map.

Now if you work according to the Globe or true Map, for the finding of the difference of Latitude, and consequently the Latitude of the

the second place, it is all one with the worke after the plaine *Chart.*

But for the difference of Longitude, it is found by the severall wayes set downe in the fourth *Pro:* where the difference from the truth is set downe: And the true difference of Longitude is thereby found to be 68 deg. which added to 309. maketh 377 deg: from whence 360 deg. being taken; leaueth 17 deg. for the true longitude of the Lizard according to the first assumption.

So that by the resolution of these questions, it may bee gathered that no two places not lying under the Equinoctiall or Meridian line, can be truly scituate in the ordinary Chart. For if you will scituate them by Latitude and Longitude, their distance will be more then it should be, and the bearing more to the East or West then it ought to be, as appeareth in the third *Pro:*

Againe, if you will scituate them by their true course and distance, keeping the Latitudes true as you ought, the difference of longitude will be lesse then it should be, as appeareth by the 4, 5, and 7 *Pro:*

And lastly, if you will scituate by Course and Distance, respecting their Longitudes, then the difference of Latitude will be more then the truth, as by the sixth *Pro:* you may perceive.

All which Errors are more grosse and apparant, the further that the two places are distant from the Equinoctiall towards either Pole.

And thus much shall suffice for the resolution of the former Questions of the Map *Arithmetically*, which who so well understandeth, may thereby be able to performe any other *Nauticall Question*, that is to be resolved upon the Map without the same, by Arithmeticall calculation onely, with the helpe of the Table of Latitudes, and the Canon of Triangles.

All which Propositions or any other questions of right Lines, or right angled Sphæricall Triangles, may be performed by the *Circular Scale* without *Arithmeticke*: The use of which Instrument is facile, and fitting for all Practitioners in the *Mathematickes.*

TWO

Two most profitable Propositions for the finding of the Variation of the Compasse.

FOr that the finding of the Variation of the Compasse is of most necessary use for the Mariners direction in Sayling; I have hereunto added two principall *Propositions*, for the finding of the true Amplitude or Azimuth of the Sunne, whereby the Variation may bee credibly found out.

THe Amplitude of the Sunne, called also the bredth of the Suns rising or setting, is the Degrees and Minutes that the Sun riseth or setteth from the true East or West point of the *Horizon*: and is alwaies of the same denomination that the Suns Declination is of.

THe *Azimuth*, is the true point of the Compasse that the Sunne is on, at any height of the Almicanter given; whereof there are severall Cases, as hereafter shall be set downe: But first for the finding of the true Amplitude by the Latitude and Declination given: *viz.*

Data { *Latitude*, *Declination*, } *North* { 50 d. 20 d. } I demand the true Amplitude of the Suns rising?

1 *As the sine of the Complement of the Latitude: to the sine of the Declination: So is the Radius; to the sine of the Amplitude.* Or to avoid division by the second briefe Rule of the 5. of *Pitiscus*.

2 *As the Radius to the Secant of the Latitude. So is the sine of the Sunnes declination, to the sine of the Amplitude.* And therefore in this *Pro:*

1 *As*

1. *As 64278. the Sine of 40 degrees, being the Complement of 50 deg. the Latitude given: To 34202. the sine of 20 degr. the Suns Declination given: So is 100000. the Radius, to 53209. the sine of 32 deg. 9 min.* for the Amplitude required, from the East Northwards: which dividing by 11 deg. 15 min. the Degrees answering to one point of the Compasse, the Quotient is two points 9 degr. 39 min. that is E. N E, and 9 deg. 39 min. to the Northwards, for the true point of the Compasse of the Sunnes rising or setting, at that time according to the *Data*. *Or,*

2. *As 100000. the Radius, to 155572. the Secant of 50 degrees, being the Latitude given: So is 34202. the sine of the Sunnes declination, to 53209. the sine of 32 deg. 9 min.* for the Amplitude demanded as before.

But if the true Amplitude were sought at the Sunne setting, then the 32 deg. 9 min. found, must be accompted from the West Northwards in this *Pro:*

And if the Declination in this case had beene given Southwards, then the Amplitude at the Suns rising would have beene found 32 degr. 9 min. from the East, Southwards: And the Amplitude at the Sunnes setting 32 deg. 9 min. from the West Southwards; And so for any other.

Thus the Amplitude being found, the *Variation* of the Compasse is the difference, betwixt that and the *Needles* Amplitude, which every Sea-man knowes how to observe.

Note, that the greater the Latitude, the greater is the Amplitude; For where the Latitude is equall to the Complement of the Declination given, both being of one denomination, the Amplitude is 90 deg. from the East or West; because there the Sunne toucheth as it were the *Horizon* at the lowest, in the intersection of the *Horizon* and *Meridian Circles*.

But where the Latitude is more then the Complement of the Declination given; both being of one denomination, that is both North, or both South: there the Sunne commeth not at all to the *Horizon*, and so in that respect cannot bee said either to rise or set; for that it is there continually Day, so long as the Declination is equall to, or more then the Complement of the Latitude?

 The

The Latitude, Declination and Almicanter of the Sunne being given: To find the Azimuth.

This Proposition hath three Cases: For,

Either the Sunne hath —— { No, North, South } *Declination.*

In all which Cases.

Adde the Complement of the Latitude A B, to the Complement of the Almicanter B C, the totall will be A F.

Also adde the Complement of the Latitude G N, to the Almicanter D G, the totall will be D N, whose sine is D P.

1. If A B and B C, bee equall to a Quadrant (as in the second Diagram) then is D T, the ½. of the sine D P.

2 If A B and B C, be lesse then a Quadrant (as in the third Diagram) the Complement of that summe is F Q; whose sine is F u, which taken from D P, the Remainder is D R, the ½. whereof is D T.

3 If A B, and B C, be more then a Quadrant (as in the first 4. and 5 Diagram (the excesse thereof is F Q, whose sine is F u, which added to D P, the whole is D R, the ½· whereof is D T.

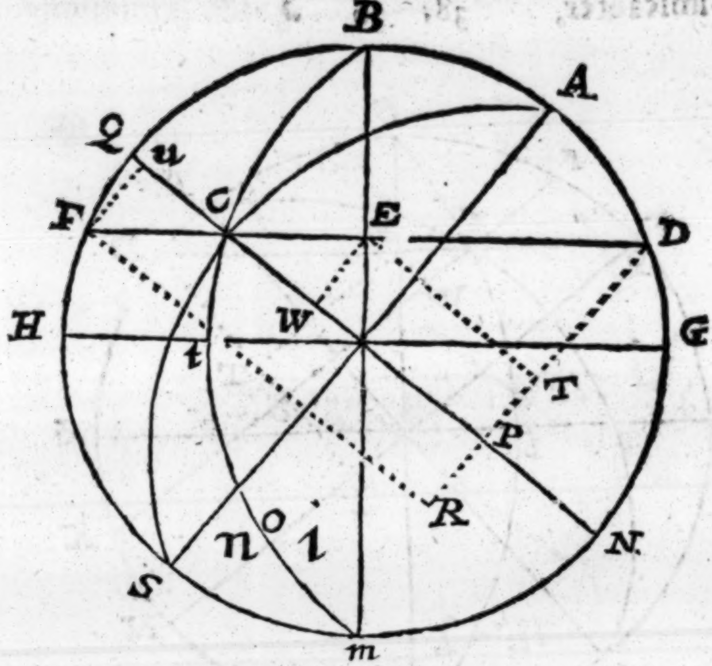
1 *Example where the Sunne is in the Equinoctiall* A B, *and* B C; *being alwayes in this case more then a Quadrant.*

Data { Latitude, 51 *deg.* 30 *m. North*; Almicanter, 20. —— —— } *Demand the Azimuth.*

Let H B G N, *be the Solsticiall Colure.*
H G, *the Horizon.*
B E M, *the East Azimuth.*
A S, *the Axis of the world.*

Q N,

QN, *the Equinoctiall.*
FD, *the Almicanter.*
IK, *(in the other Diagrams) the Sunne paralell.*
BG, *equall to* BF, *or* BD, *and consequently* Ct, *equall to* FH, *or* DG, *the Almicanter given.*



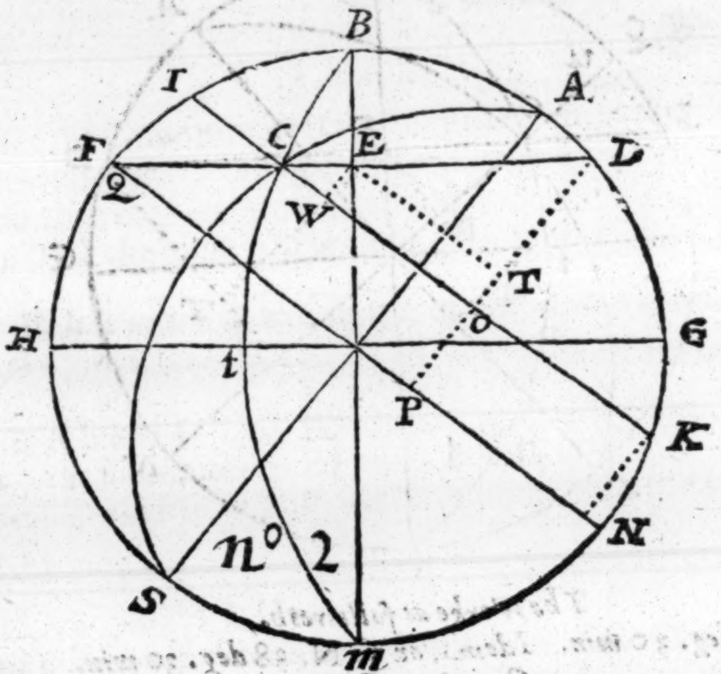
The Worke as followeth.

AB, 38 *deg.* 30 *min.* *Idem*, *or* GN, 38 *deg.* 30 *min.*
BC, 70. — *Compl:* DG, 20. --
AF, 108. 30. —— DN, 58. 30. DP, 85264
AQ, 90. ——
FQ, 18. 30. *Whose Sine is* —— —— Fu, 31730
The totall is, —— DR, 116994
½ *whereof is* RT, *or* DT, —— 58497
From whence subtract RP, *or* Fu, —— 31730
The Remainder, is PT, *or* WE, —— 26767
Then

As DT, 58497, *to* DE, 100000. *So is* WE, 26767, *to* EC, 45758. *the sine of* 27 *deg.* 14 *min. for the true Azimuth of the Sunne from the East Southwards.*

2 *Example where the Sunne hath North declination* AB, *and* BC, *being equall to a Quadrant.*

Data	Latitude *North*,	51 d.	30 m.	*Demand the* Azimuth.
	Declination *N.*	20.	—	
	Almicanter,	38.	30.	



The Worke as followeth.

AB, 38. *deg.* 30. *min.*	*Idem, or*	GN, 38 *deg.* 30 *min.*			
BC, 51. 30.	*Compl:*	DG, 38. 30.			
AQ, 90. — —		DN, 77. —	*the*		
	Sine whereof, is	— —		DP,	97437
	½. *thereof, is* PT, *or* TD,	—			48718
	Subtract Kr, *or* OP, *the Declination,*				34202
	Resteth OT, *or* WE,	— —			14516

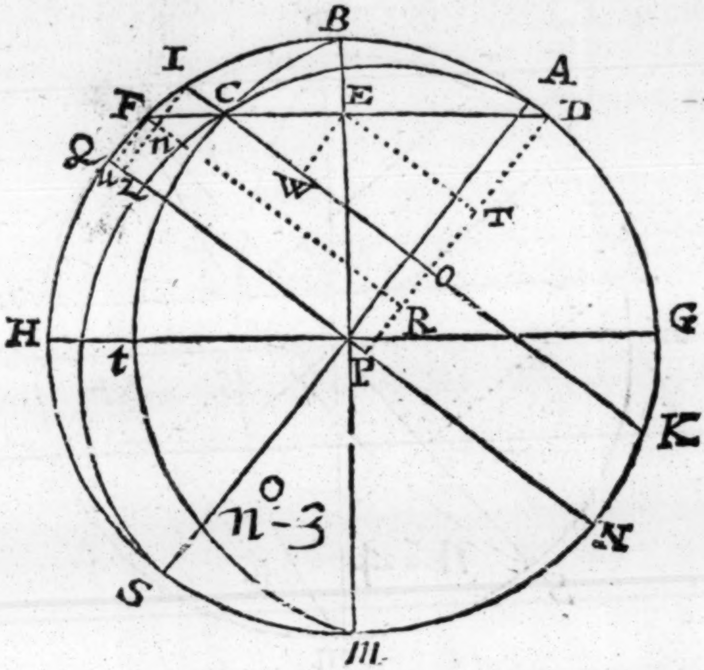
Then

As TD, 48718, *to* DE, 100000. *So is* WE, 14516, *to* EC, 29796. *the sine of* 17 *deg.* 20 *min. for the true* Azimuth *of the Sunne from the East Southwards.*

3 *Exam-*

3 *Example where the Sunne hath North declination* A B, *and* B C, *being lesse then a Quadrant.*

Data { Latitude, 51 *d.* 30 *m. North* :
Declination, 20. -- *North* :
Almicanter, 48. 30. —— } *Demand the* Azimuth.

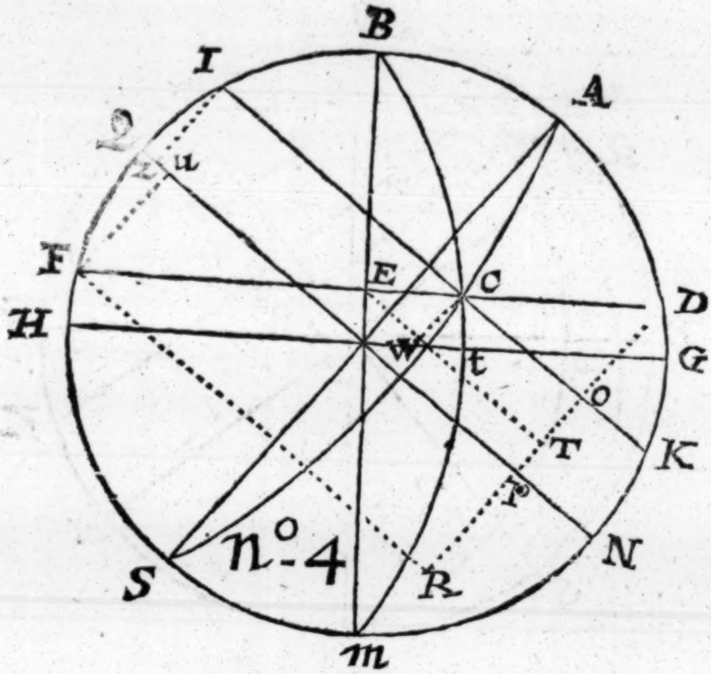


The Worke as followeth.

A B, 38 *d.* 30 *m.* *Idem, or* G N, 38 *d.* 30 *min.*
B C, 41. 30. *Compl:* D G, 48. 30.
A F, 80. -- D N, —— 87. *whose Sine* D P, 99862
F Q, 10. *whose sine is* F u, *or* P R, —— *subtracted* 17364
Resteth D R, —— 82498
½ *whereof is* T D, *or* R T, 41249
Sine of the Declination I Z, 34202 } *Resteth* I u, *or* R O, 16838
Subtract F u, —— 17364 }
Which subtracted from R T, *Resteth* O T, *or* W E, —— 24411
Then
As T D, 41249, *to* D E, 100000. *So is* W E, 24411. *to* E C, 59179.
The sine of 36 *deg.* 17 *min. for the true Azimuth of the Sun from the East Southward.*

4 *Example where the Sunne hath North declination* A B, *and* B C, *being more then a Quadrant.*

Data	Latitude, *North*: 51 d. 30 m. Declination, *North*: 20. -- Almicanter, 10. ——	*Demand the Azimuth*



The Works as followeth.

A B, 38 d. 30 m. *Idem, or* G N, 38 d. 30 min.
B C, 80. —— *Compl:* D G, 10.
A F, 118. 30. —— D N, 48 30. *Whose Sine is* D P, 74895
A Q, 90. ——
F Q, 28. 30. *Whose sine is* F u, *or* P R, *which is* —— 47715
The whole being added, is D R, —— 122610
½ *whereof is* T R, *or* T D, —— 61305

Sine of the Declination I z, 34202
F u, *being added*, —— 47715
The total is O R, —— 81917
Subtract T R, —— 61305
Resteth T O, —— *or* C W, *which is* 20612

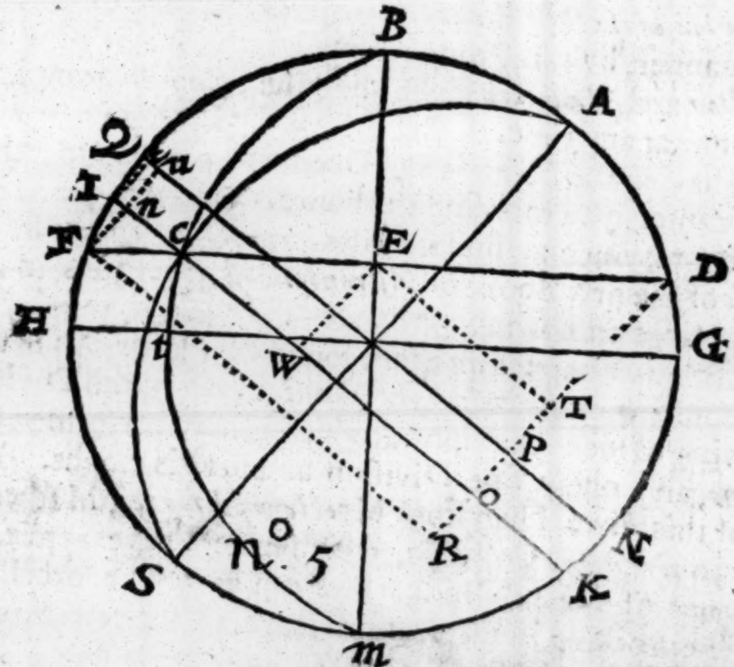
Then

Then

As T D, 61305, *to* D E, 100000. *So is* W E, 20612. *to* C E, 33622
The sine of 19 *deg.* 39 *min. for the true Azimuth of the Sun from the East Northwards.*

5 *Example where the Sunne hath South declination* A B, *and* B C, *being alwayes in this case more then a Quadrant.*

Data	Latitude 51 *d.* 30 *m. North,* Declination 10. — *South,* Almicanter, 15: —	*Demand the* Azimuth:



The Worke as followeth.

A B, 38. *deg.* 30. *min.* *Idem, or* G N, 38 *deg.* 30 *min.*
B C, 75. — *Compl:* D G, 15. —
A F, 113. 30. D N, *is* 53. 30. *Whose sine is* D P, 80385
A Q, 90. —
F Q, 23. 30. *Whose sine is* F u, *or* P R, *which is* — 39874
The total is D R, — 120259
½ *thereof, is* R T, *or* T D, — 60129

The

The sine FN, *is* —— 39874 } RT, —— 60129
The sine of the Declin: 12, *subtract*, 17364 }
Resteth FN, *or* RO, —— 22510. *Subtract* 22510
Resteth OT, *or* WE, 37619

Then. *As* TD, 60129. *to* DE, 100000. *So is* WE, 37619. *to* EC, 62556. *which is the sine of* 38 *deg.* 43 *min. for the true Azimuth of the Sunne from the East Southwards*; So that having at the same time observed the Needles *Azimuth*, by comparing that with the true *Azimuth*, the difference betwixt those two numbers sheweth the *variation of the Needle or Compasse* at the time of observation: In like manner, by this Proposition, you may (having the *Latitude Declination*, and *Azimuth* given) find the *Almicanter* or depression of the Sunne at any time.

Or by having the Latitude and Declination given, together with the Almicanter, you may find the houre. Or with the said *Data* and the houre, you may find the Almicanter, as by the 8. and 16. *Problems* of the first Booke of *Astronomicall Questions* in *Pitiscus*, is more at large set downe.

All which hath heretofore beene found very laborious in the operation, requiring many workes of Multiplication and Division for the resolving of such Questions; which is now performed by *Prosthaphæricis*, and onely one Division as afore is taught. The first ground of this *Pro:* I had from Mr. *Henry Brigges*, Mathematicall Lecturer in *Gresham* Colledge, which since I have applied to the 4.th *Axiome* of the 4.th Booke of *Pitiscus*: The excellency of which briefe working in this kind, I leave to the consideration of the studious Mathematician.

Much more might have been added by way of application to this generall *Trigonometria*, out of *Pitiscus* himselfe, *Regiomontanus*, *Copernicus*, *Clavius*, *Finkius*, and others; But because my time will not now permit me, I will deferre the same, till further occasion be offered, not doubting but in the meane space, some of our *English* Mathematicians will hereby take incouragement to publish some Workes of their owne, for the benefit of our Country, which I heartily desire, and would be right glad to see effected.

Est voluisse satis. FINIS.

Example of both Members.

Againe, let the greater Arch A B, be —— 20. deg.
The lesser arch *B C*, be —— 15 deg.
The summe or totall, is —— 35 deg.
The difference, is —— 5 deg.

The sines of the giuen arches, and their Complements as before.
The sine *A B*, 3420201. The sine *A D*, or *B E*, 9396926
The sine *B C*, 2588190. The sine *C D*, —— 9659258

Let, *A B*, be multiplied by *B C*, and *B E* by *C D*, and the products subtracted from, and added one to another. And diuided by the *Radius*.

The greater Product shall be ——	9076733	2640908
The lesser Product shall be ——	885213	0026190

The difference diuided by the *Radius*, is 8191520. being the sine of the Complement, of the summe.

And the summe diuided by the *Radius*, is 9961946. being the sine of the Complement, of the difference.

39 *These nine Problems are as it were Instruments, by whose helpe all the rest of the Sines are drawne out of the totall sine.*

The most commodious order of finding of them is thus; First, are to be found out the subtenses of the Arch, of 60. deg. 30 d. 10, d, 2, d. 1. d. 20. min. 10. m. 2. m. 1. m. 20. sec. 10, s. 2. s. *by the* 5 6, & 7. *Problems.*

Likewise the subtens of the Complement of those Arches, by the first Problem.

For this Inquisition is the most exact of all other: that you may rightly call those Subtenses, principles of the Canon of Triangles. Then out of the halfe of those subtenses, that is, out of the Sines of the arches, of 30. d. 15. d. 5. d. 1. deg. 30. m, 10 m. 5. 1. min 30 s. 10. s. 5. s. 1. sec. *together with the sines of the Complements of those Arches you shall easily find out all the Sines, by the second,* 8. *and* 9. *Problems. By the second Problem, by finding out the sine of* 2. deg *also of* 2. min. *and of* 2 sec. *or* 26. sec. *By the* 8. *and* 9. *by continually adding to the sines hitherto found, the sine of* 1. deg. *or of* 1. min. *or of* 10. sec. *or also of* 1. *second: as you would haue the Table briefe, or more ample.*

I found out the subtenses aforesaid, of 60. 30. 15, &c. after the same manner, as I haue set downe in the parts of the *Radius*, of 1000000000000000000000000000. And so I found them as followeth.

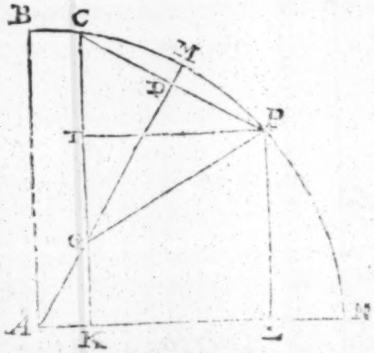
The

The Arches.			*The Subtenses.*				
60 d	00. m.	00 .s.	100000	00000	00000	00000	00000
30.	00	00	51763	80902	05041	52469	77977
10.	00	00	17431	14854	95316	34711	61285
2,	00	00	3490	48128	74567	02563	88379
1.	00	00	1745	30709	96747	86992	97569
	20	00	581	77559	68723	86874	86923
	10	00	290	88810	61083	07015	25490
	2	00	58	17764	09126	84919	27486
	1	00	29	08882	07640	17437	29548
		20	9	69627	36183	92296	70823
		10	4	84813	68106	20557	04030
		2	0	96962	73622	15273	56399

The halfe of the Arches.			*The halfe of the Subtenses.*				
30 d.	00. m.	00. s.	50000	00000	00000	00000	00000
15	00	00	25881	90451	02520	76234	88988
5.	00	00	8715	57427	47658	17355	80642
1.	00	00	1745	24064	37283	51281	94189
	30	—	872	65354	98373	93496	48884
	10	—	290	88779	84361	93442	43461
	5	—	145	44405	30541	53507	62745
	1	—	029	08882	04563	42479	63743
	00	30	014	54441	03820	07367	14774
		10	4	84813	68091	96148	35411
		5	2	42406	83053	10278	52015
		1	0	48481	36811	07636	78199

40 In this *Theorem* is an excellent Compendium of the sines. *The difference of the sines of two arches, equally distant on both sides, from 60. degrees, is equall to the sine of the distance.*

The declaration. Let C N, and P N, be the two arches equally distant from 60 d. M N, that is equally distant on both sides from the point M. And let the right lines C K, and P L, be the sines of



those arches, being perpendiculers upon the right line A N, by the 3. consect of the 7. hereof. And thereupon paralell one to another by the 38. of the first.

Moreover, let the right line P T, be drawn perpendiculer upon the right line C K, paralell to the right line K L, by the 38. of the first. This right line T P, cutteth from the right line C K, another line T K, equall unto P L, by the 39 of the first. And leaveth the right line T C, for the difference of the sines C K, and P L.

Lastly, the sines of the distance, of either of them from 60. deg. Let be the right line C D, or D P. I say, that the right line T C, is equall to the right line C D, or D P.

The Demonstration. For because in the Triangle G C P, that the perpendiculer G D, doth bisect the base C P by the 12. hereof, and by the Pro: Therefore the sides G C, and G P, are equall by the 23. of the first. And the angles C G D, and D G P, are also equall by the same; and lastly, the angles G C P and G P C, are likewise equall, by the 26. of the first. But the angle C G D, is 30. deg. for that it is equall to the angle B A M, by the 38. of the first.

Therefore the angle C G P, is 60. deg. for that it is double to the angle C G D.

But because the angle CGP, is 60. deg. therefore the other two angles G C P, and G P C, added together, are 120 deg. by the 49. of the first.

But these other two are demonstrated to be equall, therefore every of them is 60. deg.

And the angle C P G, is also so many degrees, therefore the

triangle

triangle C G P, is equiangled. But because the triangle C G P is equiangled, therefore also it is equilaterall by the 28. of the first.

Moreouer, because the triangle C G P, is equilaterall, therefore the perpendiculer P T, bysecteth the base CG, by the 23, of the first.

Then the sides C P, and C G, are equall.

Therefore also their bisegments C T, and C D, are equall, which was to be demonstrated.

Consectarie. The sines of whatsoeuer 60 degrees, being giuen, you may find the sines of the other, 30. degrees by addition or subtraction onely.

The Illustration by Numbers. Let the arches C N, be 70: deg. PN, 50, deg. C M, or P M, 10. deg. for so many degrees are the arches of 70. deg. and 50 deg. distant from the arch of 60. deg on both sides. And let first the sines of 70, deg, and 10, d. be giuen; And let the sine of 50. degrees be demanded.

From the sine of 70. degrees C K,	9396926
Subtract the sine of 10. deg. C D, or C T,	1736482
The remainder will be the sine of 50: deg. T K, or P L,	7660444

Then let the sine of 70, deg. and 50 d. be giuen, And let the sine of 10. degrees be demanded.

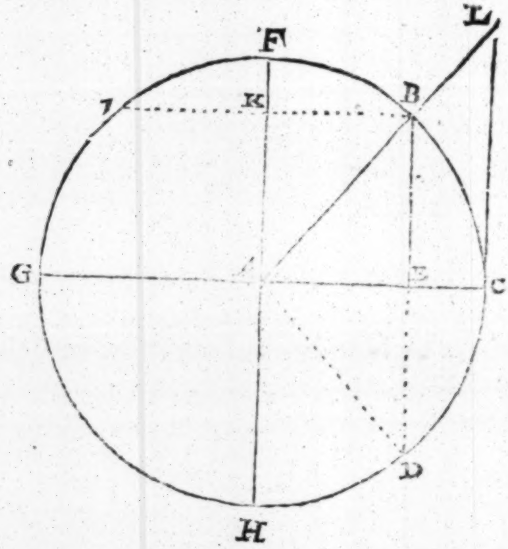
From the sine of 70. deg. C K,	9396926
Subtract the sine of 50, deg, T K, or P L,	7660444
The remainder will be the sine of 10, d. T C, or C D,	1736482

Lastly, let the sines of 50. deg, and 10, deg. be giuen. and let the sine of 70, deg. be demanded.

To the sine of 50. deg. P L, or T K,	7660444
Adde the sine of 10. deg. D P, or T C,	1736482
The whole will be the sine of 70, deg. C K,	9396926

41. *And thus farre of the making of the tables of right sines, The tables of versed sines are not needfull, as aforesaid.*

42 *The tables of Tangents and Secants, are thus made out of the tables of right sines:*



1 As the sine of the complement to the sine of an arch : So is the *Radius* to the Tangent, of that arch.

2 As the sine of the complement to the *Radius* : So is the *Radius* to the *Secant*, of that arch.

For by the 46. of the first.

1 As A E, to E B, So is A C, to C L.

2 As A E, to A B, So is A C, to A L. As for example. Let the *Tangent* and *Secant*, of the arch B C, 32. deg. be sought for : The sine of 30. deg. is 5000000. B E.

The sine of the complement, 60. deg. is 8660254. A E, Then I say.

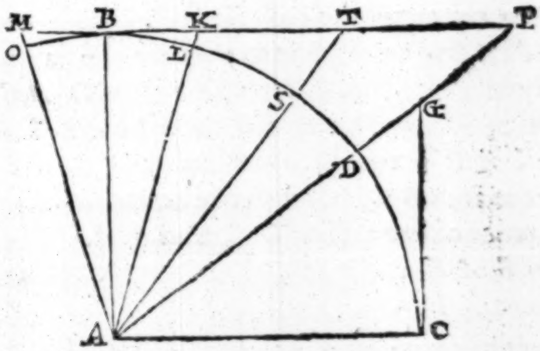
1 As A E, 8660254. to B E. 5000000: So is A C, 10000000 to C L, 5773503. Therefore the *Tnagent*, of the arch, of 30. deg. is 5773503.

2 As A E. 8660254. is to A B, 10000000. So is A C. 10000000. to A L, 11547005. Therefore the *Secant* of the arch of 30. deg. is 11547005.

43 *The briefe Rules of the* Tangents *and* Secants, *are excellent in these three Theorems following.*

The first Theorem. The difference of the Tangents, *of any two arches, making a Quadrant, are double to the* Tangent, *of the difference of those arches.*

The



The Declaration. Let the two arches making a Quadrant be C D, and B D, whose Tangents are C G, and B P.

Let B S be an arch made equall to C D, whereupon S D will be the arch of the difference, of the two given arches C D, or B S, and B D. Also let the Tangent B T, bee equall to the Tangent C G; whereupon the right line T P, will be the difference of the Tangents given C G, or B T, and B P. Lastly, let the arches B L and B O, (whose Tangents are B K, and B M,) be made equall to the arch S D; I say, that the right line T P, being the difference of the two given Tangents, C G and B P, is double to the right line B K, being the Tangent of the difference of the two given arches: Or, which is all one, I say that the right line T P, is equall to the right line M K.

The Demonstration. For if you take equall things from equall, the remainder shall be equall. But the right lines K P, and M T, are equall. Therefore if you take the right line K T, from both of them, the right lines T P, and M K remaining, shall be equall.

The assumption is proved. For those things that are equall to one and the same things, are also equall one to another.

But the right lines K P, and M T, are equall to the same right line K A. Therefore they are equall one to another.

Againe, the assumption is proved. And first, that the right line K P, is equall to the right line K A, is thus proved.

Because the angles *K A P*, and *K P A*, in the triangle *AKP*, are equall. Therefore also their opposite sides, that is the side *KA*, and *KP*, are equall by the 5. of the first. And that the angles *K A P* and *K P A*, are equall one to another, thus appeareth; for that they are equall to one and the same angle *D A C*. For the angle *K P A*, is equall to the angle *D A C*, by the 38. of the first. And the angle *K A P*, is equall to the same angle *D A C*, by construction. For the arch *B L*, is put to bee equall to the arch *S D*, being the difference of the arches *D C*, and *B D*. Therefore the angle *B A L*, or *B A K*, is the difference betwixt the angles *B A P* and *D A C*. Seeing therefore the angles *K A P*, and *K P A*, are equall to the same angle *D A C*. It followeth necessarily that they are also equall one to another.

Then that the right line *M T*, is equall to the right line *K A*, or to the right line *M A*, by the Proposition, is thus proved.

For that in the Triangle *A M T*, the angles *M T A*, and *M A T*, are equall, therefore also the sides opposite unto them, that is; the sides *M T* and *M A*, are equall by the 5. of the first.

And that the angles *M T A*, and *M A T*, are equall, thus appeareth. Because the angle *M T A*, is equall to the angle *T A C*, by the 38. of the 1. And the angle *T A C*, is equall to the angle *T A M*, by the Proposition, for the arches *C S*, and *S O*, are put to be equall. The same reason is, if the difference *B L* be greater then halfe the complement *B S*, onely the letters *L* and *S*, and also *K* and *T*, are put one for another.

Generally therefore, the Difference of the *Tangents* of two arches making a Quadrant, is double to the Tangent, of the difference of those arches which was to be demonstrated.

Consectarie. Therefore the *Tangents* of two arches, being given, making a quadrant, the Tangent also of the difference of those two arches, is also given.

And contrarily, the Tangent of the difference of these two arches being given, together with the Tangent of the one arch; the Tangent of the other arch, is also given.

The Appendix.

This *Theoreme* may also be thus propounded. The double Tangent of an arch, with the Tangent of halfe the complement, is equall to the Tangent of the arch, composed of the arch given, and

halfe

halfe the complement thereof.

For if the arch B L, bee put for the arch given, the double *Tangent* thereof shall be T P, by the demonstration before going, And the complement of the arch B L, shall bee the arch L C, whose halfe is the arch L D, or D C, whose *Tangent* is the right line G C or B T. But T P, added to B T, maketh B P, being the *Tangent* of the arch BD, composed of the giuen arch B L, and halfe the complement L D, Therefore &c.

44 *The second Theorem. The* Tangent *of the difference of two arches, making a quadrant, with the* Tangent *of the lesser arch maketh the* Secant *of the difference.*

As the *Tangent* of the difference B L, or B O, that is the right line B K, or B M, with the *Tangent* of the lesser arch D C, or B S, that is with the right line B T, maketh the right line M T, which is equall to the right line A K, being the *Secant*, of the difference B L, by the demonstraton of the first *Theorem*.

Consectarie. Therefore the *Tangent* of the difference of two arches, making a quadrant, and the *Tangent* of the lesser arch being given, the *Secant* of the difference is also given. And contrarily, &c.

The Appendix.

This *Theorem* may be also thus proponnded. *The* Tangent *of an arch with the* Tangent *of halfe the complement, is equall to the* Secant *of that arch*, For if you haue the arch B L, or B O, for the arch given, the *Tangent* of the arch giuen, shall bee B M, the *Tangent* of halfe the complement, shall be B T, which two *tangents* added together, make the right line M T, But the right line M T, is equall to the right line A K, by the demonstration of the first *Theorem* which right line AK : is the *Secant* of the arch given BL, by the propo : Therefore, &c.

45 The 3. Theorem. *The Tangent, of the difference of two arches, making a quadrant with the Secant of their difference, is equall to the tangent, of the greater arch.*

As the tangent of the arch B L, being the difference of the two arches B C, and D C, making a quadrant, with the *Secant* of the same arch B L, that is the right line B K, with the right line A K, is equall to the right line B P, by the demonstration of the first *Theorem*.

Consectarie. Therefore the *Tangent*, of the difference of two arches, making a quadrant being given, with the *Secant* of their difference, the *Tangent* of the greater arch is also given: And contrariliy, &c.

The Appendix.

This *Theorem*, may also bee thus propounded. *The Tangent of an Arch, with the Secant thereof, is equall to the Tangent of an arch composed of the arch given, and halfe the Complement?*

For if you have the arch B L, for the arch given, B K shall bee the *Tangent*, and A K the *Secant* of the arch given.

But the right lines A K, and K P, are equall by the Demonstration of the first *Theorem*; Therefore the *Tangent* of the arch given B L, that is the right line B K, with the *Secant* of the same arch, that is, A K is equall to the right line B P, which is the *Tangent* of the arch B D, being composed of the given arch, B L and L D, being half the Complement.

46 *In the Table following, you have Examples of the three precedent Theorems.*

Deg.

Deg.	Mi.	Tang. of 30 sec. is 1454	The Secants.
89	59	34377466738	34377468292
	1	2909	
		34377463829	4 By subtr.
89	58	17188731914	17188734823 By addi-
	2	5818	
		17188726096	
89	56	8594363048	8594368866
	4	11635	
		8594351413	2 By subtr.
89	54	4297175706	4297187341 By addit.
	6	23271	
		4297152435	9 By subtr.
89	44	2148571217	2148599488 By addi.
	16	46542	
		2148529675	80 By subtr.
89	28	1074264817	1074311379 By addi.
	32	93087	
		1074171750	
88	56	537085875	537178962
1	04	186190	
		536899685	3 By subtr.
87	52	268449842	268636032 By addi.
2	08	3722509	
		268077333	2 By subtr.
85	44	134038666	134411175 By addi.
4	16	746053	
		133292613	60 By subtr.
81	28	66646306	67392359 By addi.
8	32	1500458	
		65145848	
72	56	32572924	34073382
17	04	3700034	
		29502890	
55	52	14751445	17821479
34	08	6778997	
		7972448	
[illegible]	[illegible]	3986224	10765221

In which Table, first the Tangents of two arches, making a Quadrant being given; to wit, of the arch of 89 deg. 59 min. and of the arch of 1. min. The Tangent of their Difference, being 89 deg. 58 min. is found out. Moreover, by this Tangent, and the Tangent of the Complement of the arch being 2 min. the Tangent of the difference being 89 deg. 56 min. is found. And so forward, untill the Complement under-written, could not be taken out of the arch given any more, which was done unto the arch of 29 deg. 44 min. whose complement is 68 deg. 16 min. which cannot be taken from the arch 21 deg. 44 min. So then I say, that all the Tangents are found out by the first *Theorem*. Then all the Secants of the same arches, except the first are found out, by continuall adding of the Tangent of the Difference to the Tangent of the lesser arch by the second *Theorem*: or by subtracting the Tangent of the Difference from the Tangent of the greater arch, by the third *Theorem*. And the first Secant is found, by adding the Tangent of ½. the Complement, to wit; 30 *sec*. to the Tangent of the given arch, being 89 deg. 1. min. by the second *Theorem*.

But if, beyond this continuation of Examples, *viz*. by the Tangent of the arch giuen, being 21. deg. 44. min. and of halfe the complement thereof being 34. deg. 8. min. the tangent of the arch composed of the giuen arch 21. deg. 44. min. and halfe the Complement being 34. deg. 8 min. to wit, the arch of 55. deg. 52. mine. be demanded: The appendix of the first Theorem is to be vsed.

And if by the same tangents of the arch of 21. deg. 44. min, and of halfe the Complement 34. deg. 8. min. the Secant of the arch of 21. deg 44. min. were demanded: You must use the appendix of the second Theorem.

Lastly, if by the Tangent and Secant of the arch of 21. deg. 44. min. the tangent of the arch composed of the arch giuen, being 21 deg. 44 min and ½. the complement being 34. deg. 8. min. to wit, of the arch 55. deg. 52. min. were demanded: Then you must worke by the Appendix of the third Theorem.

47 *And thus much of the making of the Table: The proofe of*

the Tables now made, may be done divers wayes: viz. *Eyther by the Rules and precepts hitherto set downe for the making of the Tables, or by the first, second, and third Differences of the Sines, Tangents, and Secants.*

48 *And by what meanes this proofe may be made: It is to be understood, that howbeit some Number in the end, may seeme to bee a false Number, yet it is not a false Number.* As if you examine the *Tangents* following by the 43 hereof, after this manner.

77 deg. 49 min. the Tangent,	45045072
12 deg. 31 min. the Tangent,	2219999
	43825073
64 deg. 55 min. the Tangent,	21412536

The last Tangent 21412536. in the last Figure, doth not answer to the *Tangent* put in the *Table*, for there the last figure is 7. And yet there is no errour in these three Tangents: and the reason why the Tangent 21412536. came out in this proofe lesse just by 1. is, because the Tangent 2219999. was greater, just by 1; and therfore it is subtracted too much from the tangent 45045072. But if you put the Tangent 2219998. for the Tangent 2219999. this shall be lesser then the true Tangent, and the last Tangent 21412537. shall come for the greater then the truth. Therefore for such small difference, which by no meanes can be avoyded, the Table is not to be accompted erronious.

49 *In the other Figures, except the last, if any Errour be, It may easily bee found out by the first, second, and third Differences.* At adventure in the table, let the Tangent of 77 deg. 26 min. be taken, which is 44494381. and let it be suspected that there is some errour therein. Set downe in order some of the Tangents with their Differences, first, second, and third, after this manner.

Tangent

deg.mi	Tangents.	differ.1	diff.2.	diff.3.	deg.m.
12. 31	450450 72				77 29
12. 32	449832 21	61851			77 28
12. 33	449215 32	61689	162		77 27
12. 34	448600 04	61528	161	1	77 26
21. 35	447986 36	61368	160	1	77 25
— 36	447374 28	61208	160	0	— 24
37	446763 79	61049	159	1	— 23
38	446154 89	60890	159	0	— 22
39	445547 56	60733	157	2	— 21
40	444941 81	60375	158	0—1	— 20
41	444337 62	60419	156	2	— 19
42	443734 99	60263	156	0	— 18
43	443133 92	60107	156	0	— 17

And you ſhall partly perceive eyther by the firſt, or certainly by the ſecond differences, that the Number 4494381. in the third place, from the right hand is falſe; becauſe in the ſecond differences after 157. followeth 358, which cannot be but falſe; therefore put 158. for 358. and ſubtract that 158. from the firſt difference next afore-going, being 60573. the remainder ſhall bee in the firſt differences 60573. for the ſumme next following: which again if you ſubtract from the Tangent aforegoing 44554746. the Remainder ſhall be 44494181. for the Tangent deſired: And ſo the Error ſhall be amended, and the Numbers ſtand thus, following one after another.

—39.mi.	44554756	60733	157	2
40—	44494181	60575	158	0 — 1.
41—	44433762	60419	156	2
42—	44373499	60263	156	0
43—	44313392	60107	156	0

50 *Some men have ordered their Tables in another forme. But this which you ſee ſeemeth to me moſt convenient; Wherein the Sines Tangents, and Secants, of the arches leſſe then a halfe Quadrant, are placed in the left ſide: But the Sines, Tangents, and Secants, more then halfe a Quadrant, are placed in the right ſide, to the end that whe-*

whether the question be of an arch more or lesse then a Semi-quadrant, you may presently over against it find the complement thereof. And the Sine, Tangents and Secants, of the arches lesse then a Semi-quadrant, together with their arches downwards. But the Sines, Tangents, and Secants, of the arches greater then a Semi-quadrant, together with their arches doe increase ascending upwards by every minute, except in the first degree and in the Complement thereof, where I have also vsed one, two, or ten seconds, because otherwise the Calculation there in seconds, could not haue beene without error. Instead of the differences, I have put the proportionall part either of Minutes or of tenths of seconds, for the more ease in making the Tables, I have also added the increase, wherein the vnequall proportionall parts, doe increase either by every one, or by every tenne seconds, for the greater precisenesse.

I have taken divers Radusses for necessity, to wit, of 5.7, 8. 9. 10. 11. or 12. figures; Which variety, the skilfull Arithmetician will easily reconcile, by vsing the Radius for the worke of such magnitude as every Number set downe in the table, may answer thereunto. Which that it may presently appeare, I have every where distinguished with a point put betwixt the Sines, Tangents and Secants, made for the Radius 100000 from the rest greater then that Radius; Nay where the Radius is more then 10. figures, I have put two points betwixt, whereby the Sines, Tangents and Secants of the Radius of 10 figures, may by a mark be discovered, and knowne from the greater Sines, Tangents and Secants: Where no Number is after the point, there the Radius is onely of five Ciphers 100000. as in all Tangents, and Secants, of the last five degrees.

51 *The vse of this Table generally is thus: That you may readily find out the Sine, Tangent, and Secant of any arch or angle given, not exceeding 90 degr. together with the Sine, Tangent, and Secant of the Complement: or contrarily by the same Tables, the arch of any sine, tangent, or secant given, And so in the working of triangles, you may proceed without delay.* As if you would have the Sine, Tangent and Secant, of the arch or angle of 30. deg. or of the complement thereof: All these will be given you, in the tables according to the Radius, 10000000.

Of

Of the arch of 30 deg.	*Of the Complement.*
The Sine is — 5000000.	The Sine is — 8660254.
The Tangent is, 5773503.	The Tangent is, 17320508.
The Secant is — 11547005	The Secant is 20000000.

Contrarily : If 5773503. bee a Tangent giuen, and it bee demaunded what arch or angle answereth thereunto. The table will shew, that the arch or angle answering to that tangent, is 30. deg. And likewise in the other side of the table it will shew you, that the arch or angle of 60. deg. is the complement thereof.

52 *But if peradventure Seconds, be adioyned to the Minutes, and that you must vse them in the worke, then proceed as the examples following shall teach you.*

The first example. If the sine of 12. deg. 6 min. 23. sec, be to bee found, Take in the beginning of the tables the sine of 12. deg. 6. min. which is 2096186. Then gather by the proportionall part how much the remainder 23. sec will require : in saying,

10. sec, giues 474. parts, what shall 23. sec.

$$\begin{array}{r} 23 \\ \hline 1422 \\ 948 \\ \hline 1090|2 \end{array}$$

the fa : is 1090. parts.

Lastly, to the former giuen sine. — — 2096186.
Adde the proportionall part now found. — — 1090.

And you shall haue the sine required. — 2097276.

The second example, If the tangent of the arch of 88, deg. 51. min. 34. sec. be demaunded : First take out of the tables the tangent of 88. deg. 51. sec. which tangent according to the Radius 100000 is 4981573. Then you shall find the proportionall part, for 34 sec. after this manner.

10. sec.

10 sec.	———	12065	A
the increase	-	00059	
10. sec.	———	12124	B
10. sec.	———	12183	C
10. sec.	———	12241	D
4. sec.	———	4897	E
34. sec.	———	41269	F

This adde continually first to *A* then to *B*; thirdly to *C*, & you shal have *A B C*, for 30. sec. then if you multiply *D* by 4, and cut off the last cipher, you shall have 4897 for the other 4 sec. Now adde *A B C E* together, and you shall find F.

Adde the Tangent of 88. deg. 51. min. 4981573.

The totall, is the Tangent required, 5022842,

The third Example. If the Tangent of 89 deg. 39. m. 24 sec. were to be found. You must thus proceed:

The tangent of 89 d. 39. m. 20. s. is		16634058,	A
1. sec. is	———	13425	B
The increase is	———	00022	
1. sec. is	———	13447	C
1. sec. is	———	13469	D
1. sec. is	———	13491	E

This adde continually to *B C*, and *D*, that you may find *C D*, & *E*. Lastly, collect *A B C D E*, into one summe.

And you shall have the Tangent required, ——— } 16687890.

Or more briefly, multiply the proportionall part of 1, second by 4. and the increase by so many unities as are in the progression of 4. places, that is by 6. (for such is the progression of 4 places, 0. 1. 2. 3. which progression are 6 unites) and you shall have the same Tangent after this manner.

The Tangent is	———	96634058 A.
1. sec. is	—	13425, which multiply
by 4. is	—	53700 B.
The increase	———	00022 which multiply
by 6. maketh	—	00132 C. *Adde A B C,*

together, and you shall have — 16687890. for the Tangent desired, according to the *Radius*, 100000.

53 *But if contrarily any sine, tangent or secant were given, whose arch you would also find in seconds precisely. So proceed as the examples following will teach you.*

The first example. If 2097276 were given for a sine, the Radius being 10000000. And it were demanded what arch were answerable thereunto?

First, seeke out in the tables, the next lesser sine, and subtract that from the sine given and note the arch agreeable thereunto: Then out of the Remainder you shall collect the seconds after this manner.

The sine given is —— 2097276.
The lesser sine next unto it, is 2096186. of the arch 12 d. 6. sec.

Which subtracted, the Remainder is — 1090
10. sec. in the table is answerable to — 474
Now if 474. giue 10. sec. what shall — 1090 give?
10900
474 (2

Therefore the arch sought for, is — 948 answer 23, almost
12. deg. 6. min. 23. sec. almost 1420.
474 (3 almost
1422

The second example.

If 5022839, according to the Radius 100000. be given. And the arch answering thereunto were demaunded: First againe find in the tables the next lesser tangent, and the arch answering thereunto. Then subtract that lesser tangent, from the tangent given, and out of the Remainder you shall gather the seconds after this manner.

The tangent given, is 5022839.
The lesser tangent next 4981573. of the arch of 88. d. 51. m.
The Remainder —— 41266.
Subtract —— 12065 the parts for 10. sec.

The Remainder is — 29201. From whence

10. sec.

10 sec. gives 12069 — 29201.
The increase 00059 — 12124. 10 seconds.

10 sec. — 12124 — 17077. remaineth :- from whence
10 sec. — 12183 — 12183. the parts of 10 sec. subtracted.

10 sec. — 12242 — 4894. remaineth
1. sec. — 1224 — 1224, the parts for 01. seconds.

Now 1224 — 4896. is in 4894. almost 4. times : for foure times 1224. maketh 4896 : Therefore the arch answerable to the Tangent given, is 88 deg. 51 min. 34 sec.

The third Example.

Let the Tangent 16687890. be given, according to the *Radius* 100000. And let it bee demanded what arch is answerable thereunto : You shall proceed in this order.

The tangent giuen, is 16687890
The next lesser tangent 16634058. of the arch 89. d. 39. m. 20. s.
Which subtracted resteth 53832

The parts of 1. sec is — 13425 A.
The increase is — 00022, this adde to A, B, and C.
13447. B.
13469 C.
13491 D. Now adde A, B, C, & D.

The totall amounts to — 53832 for 4 sec. Therefore the arch answering to the Tangent giuen, is 89 deg. 39. min. 24. seconds.

54 *By this Table, after this manner, you shall be able, without any errour in the doctrine of Triangles, to worke to seconds. And in the first and last degree especially, more certainly then by* Rhæticus *his great Tables : But in all other degrees,* Rhæticus *his Tables are better; For by that you shall worke more speedily, and not onely to seconds, but also thereby you may gather the thirds and fourths exactly. Therefore if you be wise and of abilitie, be not without that Table:*

The end of the second Booke.